A Kinematic Model for Mechanical Seals With Antirotation Locks or Positive Drive Devices

I. Green

I. Etsion

Department of Mechanical Engineering,
Technion, Haifa 32000, Israel

A kinematic model of mechanical face seals is presented. Two basic seal arrangements are considered: a flexibly mounted stator with antirotation locks, and a flexibly mounted rotor with positive drive devices. The equation of kinematic constraint is derived and presented in a simple form for all the possible types of antirotation or positive drive mechanisms found in practical seals. This simple form is then used to derive the dynamic moments acting on the flexibly mounted element of the seal.

Introduction

Seal dynamics has become the subject of many investigations in the last decade [1]. Much effort is devoted to analyze the time dependent behavior of the flexibly mounted element of the seal. This element can be either the rotating one, as in many low speed applications, or the stationary one as shown in Fig. 1. Its motion is affected by factors such as axial runout, shaft vibration, dynamic properties of both the flexible support and the lubricating fluid film, etc. These factors were considered in previous works [2–4] and the existing theoretical models are quite close to realistic seals. There is, however, one aspect which has been overlooked so far, but nevertheless plays an important role in seal dynamics. This is the constraint imposed on the flexibly mounted element by positive drive devices in the case of a flexibly mounted rotor or by antirotation locks (see Fig. 1) in the case of a flexibly mounted stator. Understanding of this constraint is essential for a correct formulation of the seal kinematics. Unfortunately, there are numerous different arrangements of positive drive or antirotation devices [5, 6] e.g., dents, keys, pins, slots, and ears, and bellows to name just a few. In addition, the number of units in a particular arrangement may vary in different designs, ranging, for example, from one to four pins per seal. Manufacturing tolerances regarding these devices are fairly large and therefore even in cases where several drives or locks are present only one of them may actually be effective.

The constraint situation described above complicates any attempt to deal accurately with the kinematic model of mechanical seals. Such a model is, however, necessary for the derivation of the equations of motion. The present paper describes a general treatment that offers a fairly accurate solution to this complex problem. Based on the fact that the angular displacements of the flexibly mounted element are very small it will be shown that a first order approximation serves as a good general model, with a truncation error of order γ^2 where $\gamma << 1$. Finally the dynamic moments that act upon the flexibly mounted element will be derived for the two basic arrangements where this element is either the rotor or the stator.

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The Kinematic Model

Figure 2 presents schematically a model of a mechanical face seal which assists in understanding the kinematics of the flexibly mounted seal element. The figure shows a system of

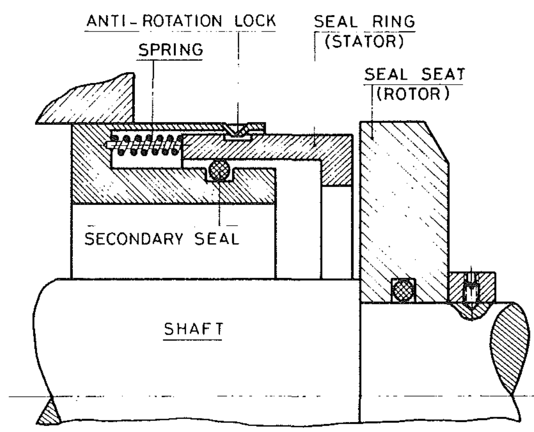

Fig. 1 Mechanical face seal-schematic and terminology

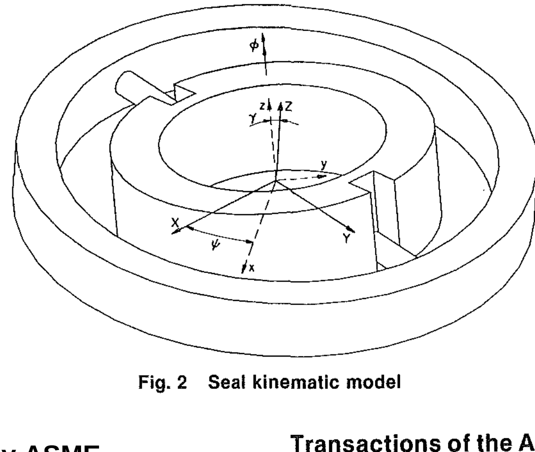

Fig. 2 Seal kinematic model

two rings. An outer ring to which a reference XYZ is attached, and an inner ring with two slots to accommodate two pins that are fixed to the outer ring along the Y axis. The inner ring represents the flexibly mounted element of the seal and is free to have two orthogonal tilts about two of its diameters.

Two cases will be considered. 1) The flexibly mounted seal element is stationary, and 2) the flexibly mounted seal element is rotating. In the first case the outer ring represents the seal housing (see Fig. 1), the two pins represent the antirotation locks, and the reference XYZ is inertial. In the second case the outer ring represents the shaft, the two pins represent the positive drive mechanism and the reference XYZ, together with the outer ring, rotates at an angular velocity ω about the Z axis.

The resultant of the two tilts of the inner ring can be described by the two Eulerian angles γ and ψ (see Fig. 2). The angle γ is the nutation of the inner ring about the axis x of a reference system xyz. This reference system is free to rotate with respect to the inner ring so that axis y is always directed to the point of maximum distance of the inner ring from the XY plane. The angle ψ is the precession of the reference xyz with respect to the reference XYZ. The axis z of the rotating xyz reference coincides with the principal axis of the inner ring. It is this axis about which the inner ring has a spin ϕ with respect to the xyz reference.

An observer located in the reference xyz sees the reference XYZ and, hence, the outer ring rotating through an angle $-\psi$ about axis Z while the inner ring rotates through an angle ϕ about axis z. The kinematic constraint forces the two rings to complete one revolution simultaneously, while any pair of corresponding points on the circumference of the two rings return to their original relative position after the completion of each revolution. This kinematic quality is characteristic of any universal joint and, hence, the seal model of Fig. 2 can be represented by a universal joint as shown in Fig. 3. Here, the rotation $\beta = -\psi$ is the input to the joint related to the outer ring, and the rotation ϕ is the output from the joint related to the inner ring.

The kinematic constraint represented by the two pins in Fig. 2 reduces the number of rotational degrees of freedom of the system into two, and dictates a certain relation between the Eulerian angles. This relation, known as the equation of kinematic constraint, has the general form

$$\phi = \phi(\gamma, \psi) \tag{1}$$

and is typical for universal joints (for a full description of universal joints see for example refs. [7] and [8]).

As stated in the Introduction, numerous different arrangements of antirotation locks and positive drive mechanisms can be found in mechanical seals. Each one of these arrangements may result in a different particular form of equation (1) making it impossible to derive a general kinematic formulation of the problem. This shortcoming can, however, be overcome by noting that the nutation angle γ in any practical seal is very small. Hence, for small γ the spin ϕ in a general joint as shown in Fig. 3 can be expanded in the form

$$\phi = \phi_0 + \phi_1\gamma + \phi_2\gamma^2 + \ . \ . \ . \tag{2}$$

where $\phi_i = \phi_i(\beta)$ are general periodic functions of β, and $\beta = -\psi$. For $\gamma = 0$ we have $\phi = \beta$, hence

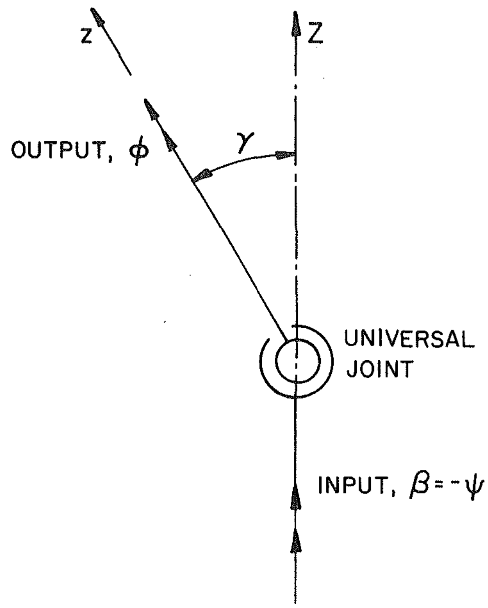

Fig. 3 Universal joint model

$$\phi_0 = \beta$$

Differentiating equation (2) with respect to time yields

$$\dot{\phi} = \dot{\beta} + \frac{\partial \phi_1}{\partial \beta}\,\dot{\beta}\gamma + \phi_1\dot{\gamma} + \frac{\partial \phi_2}{\partial \beta}\,\dot{\beta}\gamma + 2\phi_2\gamma\dot{\gamma} + \tag{3}$$

The transmission law of the joint is thus given by

$$T = \frac{\dot{\phi}}{\dot{\beta}} = 1 + \frac{\partial \phi_1}{\partial \beta}\gamma + \phi_1\frac{\dot{\gamma}}{\dot{\beta}} + \frac{\partial \phi_2}{\partial \beta}\,\gamma^2 + 2\phi_2\gamma\frac{\dot{\gamma}}{\dot{\beta}} \tag{4}$$

For $\gamma = 0$ any universal joint results in $T = 1$. Hence,

$$\phi_1 = 0$$

and the transmission law can be written as

$$T = 1 + \frac{\partial \phi_2}{\partial \beta}\,\gamma^2 + 2\phi_2\frac{\gamma\dot{\gamma}}{\dot{\beta}} + \tag{5}$$

For small perturbation, the order of $\dot{\gamma}$ is the same as that of γ, and equation (5) takes the form

$$T = \frac{\dot{\phi}}{\dot{\beta}} = 1 + 0(\gamma^2) \tag{6}$$

Hence, for any practical mechanical seal where $\gamma^2 < < 1$ equation (6) gives the transmission, T, accurately enough, by $T = 1$.

In a constant velocity joint [8] the result $T = 1$ is accurate independent of γ. This special case is characterized by the lack of preference to the order and direction of the two perpendicular tilts of the inner ring, and is typical, therefore, to such cases with axial symmetry where the pins of Fig. 2 are either omitted or are not in effect. The first case corresponds to a flexible support that consists of a metal bellows for example. The second case occurs when the friction in the elastomeric secondary seal is by itself sufficient to prevent rotation of the flexibly supported element regardless of the pins mechanism.

Nomenclature

- I = transverse moment of inertia
- I_z = polar moment of inertia
- $\mathbf{L}$ = relative angular momentum
- T = transmission law
- $\mathbf{T}$ = dynamic moment
- β = $-\psi$
- γ = nutation
- λ = absolute angular velocity
- ψ = relative precession
- ψ_r = absolute rotor precession
- ψ_s = absolute stator precession
- ϕ = relative spin
- ω = shaft angular velocity
- Ω = relative angular velocity
- ω_c = reference angular velocity

Subscripts

- r = rotor
- s = stator

As another example let us examine the case of a Hooke joint. The equation of kinematic constraint for this particular joint is (see p. 272 in reference [7]):

$$\tan\phi = \tan\beta\cos\gamma \tag{7}$$

Differentiating with respect to time and substituting $\tan\phi$ from equation (7), gives after some algebra

$$\dot{\phi} = \frac{\dot{\beta}\cos\gamma - \dot{\gamma}\sin\gamma\sin\beta\cos\beta}{1 - \sin^2\beta\sin\gamma} \tag{8}$$

For small nutation angles, $\gamma^2 < < 1$, we may use

$$\sin\gamma = \gamma$$

$$\cos\gamma = 1 - \frac{\gamma^2}{2}$$

so that

$$(1 - \sin^2\beta\sin^2\gamma)^{-1} = 1 + \gamma^2\sin^2\beta$$

Substituting these relations in equation (8) and neglecting terms of order higher than γ^2, will finally give

$$T = \frac{\dot{\phi}}{\dot{\beta}} = 1 + \left(\sin^2\beta - \frac{1}{2}\right)\gamma^2 - \frac{\dot{\gamma}\gamma}{\dot{\beta}}\sin\beta\cos\beta \tag{9}$$

Comparing corresponding terms in equations (9) and (4), we have

$$\phi_1 = \frac{\partial\phi_1}{\partial\beta} = 0$$

and

$$\frac{\partial\phi_2}{\partial\beta} = \sin^2\beta - \frac{1}{2}$$

giving

$$\phi_2 = -\frac{1}{2}\sin\beta\cos\beta$$

The particular form of equation (2) for a Hooke joint is therefore

$$\phi = \beta - \frac{\gamma^2}{2}\sin\beta\cos\beta \tag{10}$$

Recalling that $\beta = -\psi$ and hence $\dot{\beta} = -\dot{\psi}$ we see that for $\gamma^2 < < 1$, the Hooke joint gives the spin $\dot{\phi}$, accurately enough, by

$$\dot{\phi} = -\dot{\psi} \tag{11}$$

The constant velocity joint together with the Hooke joint cover a wide variety of antirotation locks and positive drive mechanisms. It may be concluded, therefore, that the approximation $T = 1$ and, hence, equation (11) hold for any practical mechanical seal where $\gamma^2 < < 1$. This is very important from a practical point of view since it enables a unified treatment and representation of the various practical arrangements by a common kinematic model.

It also enables the use of this common kinematic model in order to derive the general dynamic moments for any type of mechanical seal.

Another important result of the preceding analysis is that, contrary to the common belief, the kinematic constraints do not, in general, prevent momentary spin, $\dot{\phi}$, as is clearly shown in equation (3). Relative spin is completely eliminated only when $\gamma = 0$, or in the case of a constant velocity joint. The effect of the kinematic constraints is therefore not to prevent relative spin but rather to force the flexibly mounted element of the seal to return after each revolution to its original relative position with respect to its holding member (housing or shaft).

The Dynamic Moments

The term "dynamic moments" is used to describe the contribution of the inertia of a body to its behavior in the angular degrees of freedom. The rotational equations of motion of a body are formed by equating the dynamic moments with the "applied moments" that are contributed by external forces acting on the body. The correct formulation of the dynamic moments that act upon the flexibly mounted element of the seal is, therefore, essential for any dynamic analysis. This formulation will now be presented for the two basic arrangements, the kinematics of which was analyzed in the previous section.

The general form of the dynamic moment vector of a rigid body expressed in a moving reference can be found in several texts e.g., [7[, and is given by

$$\mathbf{T} = \frac{\partial\mathbf{L}}{\partial t} + \omega_c x\mathbf{L} + \mathbf{r}_{g0} \times m\mathbf{a}_0 \tag{12}$$

where **L** is the relative angular momentum vector of the rigid body defined as

$$\{L\} = [I]\{\lambda\} \tag{13}$$

and λ is the absolute angular velocity vector of the body. The vector ω_c is the rotational velocity of the reference system accelerating at $\mathbf{a}_0$, and $\mathbf{r}_{g0}$ is the location of the center of mass of the body (in our model $\mathbf{r}_{g0} = 0$). The absolute angular velocity λ of the body is given by

$$\lambda = \omega_c + \mathbf{\Omega} \tag{14}$$

where $\mathbf{\Omega}$ is the angular velocity vector of the body relative to the rotating reference.

In the seal model shown in Fig. 2 the body is the inner ring and the rotating reference is the *xyz* reference. Hence, due to the kinematic constraint $\mathbf{\Omega}$ is always along the *z* axis and by definition is

$$\mathbf{\Omega} = \dot{\phi}\hat{z} \tag{15}$$

The spin $\dot{\phi}$ is related to the precession $\dot{\psi}$ by the equation of the kinematic constraint, which for small nutation γ is given in equation (11).

Flexibly Mounted Stator. The angular velocity of the rotating reference for this case is (see Fig. 2)

$$\omega_c = \dot{\gamma}_s\hat{x} + \dot{\psi}_s\sin\gamma_s\hat{y} + \dot{\psi}_s\cos\gamma_s\hat{z} \tag{16}$$

Where the subscript *s* is used to indicate the stator as the flexibly mounted element. Substituting equations (16), (15), and (11) in equation (14) we have for the angular absolute velocity of the stator

$$\lambda_s = \dot{\gamma}_s\hat{x} + \dot{\psi}_s\sin\gamma_s\hat{y} + \dot{\psi}_s(\cos\gamma_s - 1)\hat{z} \tag{17}$$

Hence, by equation (13) the relative angular momentum vector **L** is

$$\mathbf{L}_s = I\dot{\gamma}_s\hat{x} + I\dot{\psi}_s\sin\gamma_s\hat{y} + I_z\dot{\psi}_s(\cos\gamma_s - 1)\hat{z} \tag{18}$$

where I_z is the polar moment of inertia, and $I = I_x = I_y$ is the transverse moment of inertia of the flexibly mounted element.

Using equations (18) and (16) in equation (12), recalling that $\mathbf{r}_{g0} = 0$, and that we are dealing with small angles γ so that $\cos\gamma_s = 1$ and $\sin\gamma_s = \gamma_s$ are valid approximations, we have the dynamic moments in the form

$$T_x = I(\ddot{\gamma}_s - \dot{\psi}_s{}^2\gamma_s) \tag{19a}$$

$$T_y = I(\ddot{\psi}_s\gamma_s + 2\dot{\psi}\dot{\gamma}_s) \tag{19b}$$

$$T_z = -I_z(\dot{\psi}_s\dot{\gamma}_s\gamma_s + \ddot{\psi}_s\gamma_s^2/2) + 0(\delta_s^2) \tag{19c}$$

As can be seen from equation (19*c*) the dynamic moment T_z is of order γ^2 and hence, can be neglected in any practical seal.

Flexibly Mounted Rotor. In this case the outer ring in Fig. 2 which represents the shaft has an angular velocity ω. This velocity when added to the relative precession $\dot{\psi}$ of the rotating reference *xyz* gives the absolute precession of the rotor, $\dot{\psi}_r$, in the form

$$\dot{\psi}_r = \dot{\psi} + \omega \quad (20)$$

where the subscript r is used to indicate the rotor as the flexibly mounted element.

From equations (11) and (20) we have

$$\dot{\phi} = \omega - \dot{\psi}_r \quad (21)$$

The angular velocity ω_c of the reference XYZ is given by equation (16) where the subscript s is replaced everywhere by the subscript r. Similarly, the absolute angular velocity of the inner ring is given by equation (14) where Ω is given in (15) and $\dot{\phi}$ in (21).

Hence,

$$\lambda_r = \dot{\gamma}_r \hat{x} + \dot{\psi}_r \sin\gamma_r \hat{y} + [\dot{\psi}_r(\cos\gamma_r - 1) + \omega]\hat{z} \quad (22)$$

The relative angular momentum vector **L** is

$$\mathbf{L}_r = I\dot{\gamma}_r \hat{x} + I\dot{\psi}_r \sin\gamma_r \hat{y} + I_z[\dot{\psi}_r(\cos\gamma_r - 1) + \omega]\hat{z} \quad (23)$$

and the dynamic moments have, by (12), the form

$$T_x = I(\ddot{\gamma}_r - \dot{\psi}_r^2 \gamma_r) + I_z \omega \dot{\psi}_r \gamma_r \quad (24a)$$

$$T_y = I(\ddot{\psi}_r \gamma_r + 2\dot{\psi}_r \dot{\gamma}_r) - I_z \omega \dot{\gamma}_r \quad (24b)$$

$$T_z = -I_z(\dot{\psi}_r \dot{\gamma}_r \gamma_r + \ddot{\psi}_r \gamma_r^2/2) + 0(\delta_s^2) \quad (24c)$$

Here again T_z is of order γ^2 and can be neglected in practical seals.

Summary and Conclusion

The kinematic model of mechanical face seals was presented. Two basic seal arrangements were considered. These are the flexibly mounted stator and the flexibly mounted rotor. The kinematic constraint provided by the antirotation locks in the first arrangement or by the positive drive devices in the second was shown to be similar to that of a universal joint. It was shown that in spite of the numerous variations of antirotation locks and positive drive mechanisms found in mechanical seals, it is possible to present the equation of kinematic constraint in the simple form

$$\dot{\phi} = -\dot{\psi}$$

This unified relation is the result of the very small nutation, γ, in practical seals, and is accurate to an order γ^2 where $\gamma << 1$.

The simple general form of the equation of kinematic constraint enables one to derive the dynamic moments that act on the flexibly mounted seal element. These moments are presented in equations (19) for the case of a flexibly mounted stator, and in equations (24) for the case of a flexibly mounted rotor. In both cases the dynamic moment T_z which is the axial component of the moment vector was found negligible. The two other components, namely, T_x and T_y depend on the transverse moment of inertia, I, in the case of a flexibly mounted stator, and on both the transverse and polar moments of inertia, I and I_z, in the case of the flexibly mounted rotor. The contribution of the polar moment of inertia in seals with flexibly mounted rotor alters the dynamic moments T_x and T_y as compared to the flexibly mounted stator case. This is equivalent to altering the inertia of the flexibly mounted element and may affect the dynamic behavior.

The analysis presented in this paper assumed no more than two pins as a representation of the constraint provided by the antirotation locks or positive drive devices. If three or more units are effective, then the inner ring is actually "locked" and is unable to track angular misalignment of the rigidly mounted element. Such a condition can be avoided by limiting the number of antirotation locks and positive drive devices in a seal to two units at most.

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