Proceedings of the ASME/STLE 2009 International Joint Tribology Conference
IJTC2009
October 19-21, 2009, Memphis, Tennessee, USA

IJTC2009-15106

AN FFT DETERMINISTIC SIMULATION OF ELASTIC ROUGH SURFACES IN THREE-DIMENSIONAL CONTACT AND MODEL ANALYSIS

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ABSTRACT

For practicing engineers in industry it is important to have closed-form, easy to use equations that can be used to predict the real contact area, and relate it to friction, wear, adhesion, and electrical and thermal contact resistance. There are quite a few such models in the literature, but their agreement or their effectiveness has not been determined. This work will use several measured surface profiles to make predictions of contact area and contact force from many elastic contact models and compare them to a deterministic FFT based rough surface contact model. The results show that several of the models show good quantitative and qualitative agreement despite having very different mathematical foundations.

INTRODUCTION

The ability to accurately predict real contact area as a function of load for rough surface contacts is a very difficult task due to the complex nature of real surfaces. Many models have been proposed over the years for the prediction of the real area of contact between rough surfaces. One of the very first of these was by Archard[1], who showed that although single asperity contact might result in a nonlinear relation between area and load, by incorporating multiple scales of roughness the relationship becomes linear. Archard used a concept where spherical asperities were stacked upon each other, each with smaller radii of curvature. This stacked type model was largely abandoned when the Greenwood and Williamson (GW) model[2] was published. This work presents for the first time an exact closed-form solution to the popular GW model.

Later researchers considered the multiple scales of asperities present on surfaces needed and so fractal methodologies were formulated[3, 4]. Fractal models were found to have their own deficiencies so a few other methods that consider multiple scales of roughness were also created[5-7]. For example, in the Majumdar and Bhushan (MB) fractal model[3] the contact area is calculated from truncation which predicts less plastic deformation with higher load. One correction was offered by Morag and Etsion[8]. The current work also finds that the MB model has difficulties also in the elastic range where it can actually predict negative contact forces for positive contact areas.

A handful of these models will be implemented in the current work and compared to an FFT based deterministic elastic contact model [9]. The hope is that the effectiveness of these models can be evaluated lending confidence in their applicability. Other comparisons have also been made, but not always by using data measured from a real surface and not considering as many contact models as are in the current work.

METHODOLOGY

The current work will make comparisons between the following rough surface contact models:

1. Kato and Stanley FFT deterministic elastic model [9]
2. GW closed-form solution model (with Gaussian distribution) – an exact closed-form solution is included herein.
3. Bush, Gibson, and Thomas statistical model[10]
4. Greenwood elliptical model[11]
5. Persson diffusion contact model [12]
6. Stacked multiscale model [13]
7. MB fractal (elastic part) [3]

For conciseness details for each model are omitted, but shall be discussed in a subsequent paper.

GW Closed Form Solution

This work adheres to the definitions and nomenclature of CEB [14] and Etsion and Front [15], and the reader is referred to that work. Therefore, $\beta=\eta R\sigma$, where η is the areal density of asperities, R is the asperity radius of curvature, and σ is the standard deviation of surface heights. Further, $h^*=h/\sigma$ is the dimensionless mean separation, σ_s/σ is the dimensionless standard deviation of asperity heights, and y_s/σ is the dimensionless distance between the means of asperity and surface heights. Following Green [16] the "average" elastic contact pressure is $p_e=F/A_n$, where F is the total external force (or load), and A_n is the nominal (or apparent) contact area. The pressure is further normalized by the equivalent modulus of elasticity. Likewise, the elastic real area of contact, A_r, is normalized by the nominal area, A_n. Both are given, respectively, by,

$$\frac{p_e}{E} = \frac{4}{3}\beta\left(\frac{\sigma}{R}\right)^{1/2} I_{ep} \tag{1a}$$

$$\frac{A_e}{A_n} = \pi\beta\, I_{ea} \tag{1b}$$

where

$$I_{ep} = \int_a^{\infty}\left(z^* - a\right)^{3/2} \varphi\left(z^*\right) dz^* \tag{2a}$$

$$I_{ea} = \int_a^{\infty}\left(z^* - a\right) \varphi^*\left(z^*\right) dz^* \tag{2b}$$

The integrals contain a lower bound having the following definition $a = h^* - y_s^*$. The Gaussian distribution ($\varphi^*(z^*)$) is given in many previous works. The integrals of Eqs. (2) have been considered complex and thus have traditionally been either approximated or integrated numerically. This work presents for the first time an exact closed-form solution for these integrals. For conciseness only the final results of the closed-form solutions are given. The solution is given in the Appendix. Substitution of Eqs. (A1 and A2) into Eqs. (1) provides the desired solution to the GW model.

Stacked Multiscale Model

The stacked multiscale model[13] is a simplified version of the full multiscale model by Jackson and Streator [6]. Assuming that all the frequencies of roughness are present everywhere on the surface, than the parts of the surface in contact must overcome the pressure required to flatten all the contributing frequencies or scales of asperities. This of course assumes that the contact areas are larger then this limiting frequency. Then the real contact pressure is defined by the pressure required to obtain complete contact at the scale with the largest ratio between amplitude and wavelength ($B=\Delta/\lambda$). Johnson et al. [23] provide the pressure to cause complete contact between sinusoidal surfaces as:

$$p^* = \sqrt{2}\pi E'\Delta f \tag{3}$$

where Δ is the amplitude and f is the frequency of the sine wave describing the shape of the surface (inverse of λ). By assuming that each asperity scale can be described by a sinusoidal shaped surface, the real contact pressure (p_r) is then approximated by Eq. (3), although Jackson et al. [13] have found that in some cases the pressure is slightly less.

Now consider the stacked surface originally described by Archard. As each scale is iteratively included in the model, the contact area will reduce. For self affine surfaces, the value B will continue to increase as the scale is decreased, ultimately resulting in the contact area reducing to zero[7]. However, not all surfaces may be self-affine, and if so, the iteratively increasing pressure will eventually overcome the pressure required to obtain complete contact (given by Eq. (3). Then the area of contact is given by[13]

$$A_r = \frac{F}{\sqrt{2}\pi E' B_{\max}} \tag{4}$$

Elastic part of Majumdar and Bhushan Fractal Model

Majumdar and Bhushan[3] derived a very popular model for the contact between fractal surfaces. First they assumed that the surface can be described by the Weierstrass Mandlebrot function and the fractal parameters G and D. Contact was then assumed to occur at where the fractal surface would truncate an opposing flat surface. For elastic contact the load was then calculated by elastic Hertz contact and the average radius of curvature of the parts of the surface in contact (predicted by truncation). The resulting elastic portion of the equation is:

$$\frac{F}{EA_n} = \frac{4\sqrt{\pi}}{3}\frac{D}{(3-2D)}\left(\frac{G}{\sqrt{A_n}}\right)^{(D-1)}\left(\frac{(2-D)}{D}\frac{A_r}{A_n}\right)^{(3-D)/2} \tag{5}$$

However, then a problem with this equation becomes apparent. Note that D ranges between 1 and 2. When D is greater than 1.5, Eq. (5) becomes negative, which is impossible. Therefore the elastic portion of the MB fractal is invalid. The real surfaces examined here do provide a value of D>1.5, and because of this, we will not make further comparisons with the MB fractal model in the current work.

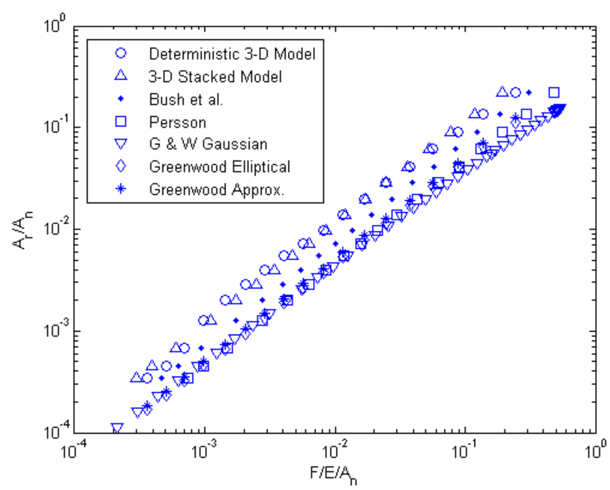

Figure 1: A comparison of real area of contact predictions by several elastic rough surface contact models for the same surface.

RESULTS

Three-dimensional surface data is recorded using a XYRIS4000CL Taicaan Con-focal Laser Profilometer over an area 1 mm^2 with a resolution of 1001x1001 points (vertical resolution is 10nm). Three surfaces with very different roughness are measured. Using the surface data, the models listed above are then used to predict the real area of contact and real contact pressure as a function of contact force. The results for one of the surfaces are shown in Figs. 1 and 2. As shown, the predictions are all within the same order of magnitude. This is actually surprising considering that the methods are derived very differently and that except for the deterministic model, reduce the surface data down to two or three parameters.

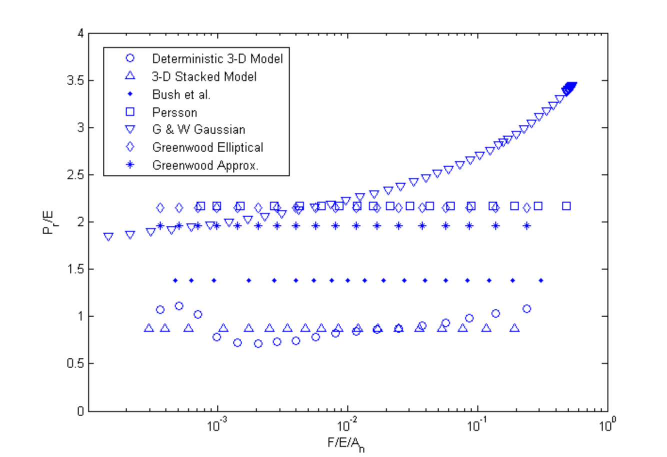

Figure 2: A comparison of real contact pressure predictions by several elastic rough surface contact models for the same surface.

ACKNOWLEDGMENTS

Thanks to XYRIS4000CL Taicaan and John McBride at the University of Southampton for lending the Taicaan Con-focal Laser Proflometer used to make the measurements, and to the DoD MURI Grant N00014-04-1-0601, Dr. P. Peter Schmidt serves as Program Officer.

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Appendix

Using the definitions $\alpha = a/\sigma_s^* \ and\ \theta = \alpha^2/4$,

$$I_{ep} = \begin{cases} \sigma_s^* \sqrt{a} e^{-\theta} \left[(1+\alpha^2) K_{1/4}(\theta) - \alpha^2 K_{3/4}(\theta) \right] \Big/ \left(4\sqrt{\pi} \right) & for \quad a > 0 \\ \Gamma(5/4) \left(\sigma_s^* \right)^{3/2} \Big/ \left(2^{1/4} \sqrt{\pi} \right) & for \quad a = 0 \\ \left(\sigma_s^*/4 \right) \sqrt{-a} e^{-\theta} \sqrt{\frac{\pi}{2}} \left[(1+\alpha^2) I_{-1/4}(\theta) + (3+\alpha^2) I_{1/4}(\theta) + \alpha^2 \left(I_{3/4}(\theta) + I_{5/4}(\theta) \right) \right] & for \quad a < 0 \end{cases} \tag{A1}$$

$$I_{ea} = \sqrt{\frac{1}{2\pi}}\ \sigma_s^*\ e^{-\alpha^2/2} - \frac{a}{2} erfc\left(\frac{\alpha}{\sqrt{2}} \right) \tag{A2}$$

where $\Gamma(\cdot)$ is the Gamma function, $I(\cdot),\ and\ K(\cdot)$ are the modified Bessel functions of the first and second kinds, respectively, and $erfc(\cdot)$ is the complementary error function.