J. Wileman

M. Choudhury

I. Green

The George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405

Computation of Member Stiffness in Bolted Connections

The member stiffness in a bolted connection has a direct influence upon safe design with regard to both static and fatigue loading, as well as in the prevention of separation in the connection. This work provides a simple technique for computing the member stiffness in many types of bolted connections. Finite element analyses are performed for joints having a range of materials and geometries, and the results are generalized by nondimensionalization. An exponential expression for the stiffness is determined, and the results are compared with those of some of the techniques currently used.

Introduction

When designing a bolted connection, e.g., Fig. 1(a), it is important to know what portion of the working load acting upon the joint must be supported by the bolt. Initial tightening of the connection induces a tensile preload in the bolt while compressing the bolted members. If an external tensile load is then applied, the connection will lengthen, and the reduction in the compressive load upon the members will equal some fraction of the external load. The remainder of the external load will result in an increase in the tensile load which the bolt must support. The manner in which the external load is divided depends upon the relative magnitudes of the spring constants, or stiffness constants, of the bolt and the members. These constants, usually referred to simply as stiffnesses, are defined for both the bolt and the members as the ratio of the load applied along the joint centerline to the resulting deflection in the same direction.

It is important to the present analysis to distinguish between the stiffness of the bolt and that of the members. The stiffness of the shank of the bolt is easily computed because the shank is essentially a bar in simple tension. The effects of the nut and the washer, as well as those of the head of the bolt, can be included in the bolt stiffness rather than in the member stiffness, as shown by Sawa and Maruyama (1976), although these effects are frequently neglected.

The behavior of the bolted members is much more complex than that of the bolt shank, so that the stiffness of the members can only be approximated. Since it is crucial to the accuracy and safety of the joint design, however, much attention has been focused upon methods of estimating this stiffness. Most of the techniques currently used to compute member stiffness are based upon rather arbitrary assumptions. One of the most frequent of these assumptions is that the stress induced in the members is uniform throughout a region surrounding the bolt hole with zero stress outside this region. Thus, a discontinuity in the stress field occurs at the boundary of this region. Rötscher first proposed this assumption (as reported by Fritsche, 1962), assuming that the stresses were contained within two conical frusta symmetric about the midplane of the joint, each having a vertex angle of 2α, Fig. 1(a). Rötscher then chose $\alpha = 45$ deg and computed the stiffness by replacing the frusta with a cylinder having the same sectional area, obtaining

$$k_m = \frac{\pi E}{4L}\left[\left(d_w + \frac{L}{2}\right)^2 - d^2\right] \tag{1}$$

where k_m is the combined stiffness of the members, L is the grip length of the joint, d is the bolt diameter, and d_w is the diameter of the washer.

Ten Bosch, Bach, and Findeisen (as reported by Stuck, 1968) relax Rötscher's assumption by requiring only that the stresses remain uniform throughout planes perpendicular to the axis, allowing them to vary in the axial direction. Thus, the axial deflection at each of these planes could be computed and the deflections integrated along the length of the axis to determine the total deflection of the conical envelope. This deflection is in turn used to compute the stiffness. When Rötscher's 45 deg angle is used, this technique results in

$$k_m = \frac{\pi E d}{2\ln\left\{\dfrac{(d_w + L - d)(d_w + d)}{(d_w + L + d)(d_w - d)}\right\}} \tag{2}$$

Shigley and Mitchell (1983) simplify this computation by assuming that the compressive load on the members is applied by means of a washer having a diameter, corresponding to bolt standards, of one and one half times that of the hole; i.e., $d_w = 1.5d$. Thus,

$$k_m = \frac{\pi E d}{2\ln\left(5\,\dfrac{L + 0.5d}{L + 2.5d}\right)} \tag{3}$$

Shigley and Mischke (1989) point out that the angle α may be left as a variable during the integration process, resulting in the expression

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$$k_m = \frac{\pi E d \tan \alpha}{2 \ln \left\{ \frac{(L \tan \alpha + d_w - d)(d_w + d)}{(L \tan \alpha + d_w + d)(d_w - d)} \right\}} \quad (4)$$

Ito et al. (1979) propose that the proper value for α depends upon the material and provide a table of suggested values based upon ultrasonic measurements of the pressure distribution at the interface. Shigley and Mischke recommend that $\alpha = 30$ deg be used, and after substituting $d_w = 1.5d$ they obtain

$$k_m = \frac{0.577 \pi E d}{2 \ln \left(5 \frac{0.577L + 0.5d}{0.577L + 2.5d} \right)} \quad (5)$$

Motosh (1976) provides the most realistic technique by allowing the stress in the members to vary in both the axial and radial directions. He assumes the stress in any plane perpendicular to the axis is maximum at the hole diameter and decreases continuously to zero at the boundary of either a conical or a spherical envelope. The compressive stress in the members is described by a fourth order polynomial depending upon r, z, and α, and the stiffness is then computed using a series of tedious numerical integrations. This method is not commonly used and is probably too unwidely for routine joint design.

Maruyama et al. (1975) perform an axisymmetric finite element analysis of a specific connection geometry, including representations of the bolt and nut deflection, and also conduct experiments using this geometry, demonstrating reasonable agreement between the finite element and experimental results. Their work indicates that the member stiffness depends upon the ratio of the member diameter to the bolt diameter. Grosse and Mitchell (1988) perform a very detailed analysis of a single-bolt joint, also restricted to a particular geometry, and note that the stiffness of the entire joint, including bolt, nut, and member deflection, is a strong nonlinear function of the externally applied load. Neither of these works includes either a general stiffness expression or data which could be used to determine stiffnesses for joints which do not match the particular geometries considered in their analyses. However, Maruyama's experimental data are useful as a validation of the expression developed in this work. By providing a numerical example based upon his data, we shall demonstrate that Eq. (9) of this work predicts Maruyama's experimental results more accurately than do his own numerical analyses.

Many factors too complex to include in the design process have been noted to affect the stiffness of the members. Both Ito (1979) and Thornley (1965) have noted the effect of surface finish upon the pressure distribution at the interface between the members, although Thornley notes that the effect can be mitigated if the preload of the joints is sufficient to cause plastic deformation of surface asperities. Finally, several of the effects of joint assembly depend upon factors which vary randomly from one joint to another and for which no general analysis techniques exist. These include asymmetry, end effects, and thread friction.

The arbitrary assumptions required by the techniques described above are violated in a practical joint, and large safety factors are frequently required to compensate for the resulting inaccuracies. The intent of this work is to present a stiffness computation technique with which these assumptions may be entirely avoided. Using finite element analysis, a dimensionless expression is derived which allows the member stiffness of many types of joints to be computed directly from the joint geometry and material properties.

Analysis Using the Finite Element Method

It is unnecessary to assume that the member stress is contained within a well-defined region about the bolt hole if the finite element method is employed. Gould and Mikic (1972) applied this technique to member deflections to determine where separation occurs at the member interface, but they did not consider the stiffnesses of the members. The finite element analyses for this work were performed on a CDC Cyber 990 mainframe using the code ANSYS[1].

In constructing the bolted joint model used in this work, two assumptions were employed so that the analysis could be conducted with reasonable computational expense. The joint was assumed to be perfectly axisymmetric, a reasonable approximation of most practical connections. Additionally, the analysis was limited to joints in which both members are of the same material and in which slippage does not occur at the interface between these members. This no-slip requirement is always satisfied in joints which have members of equal thickness (and, thus, symmetric deflections), but where the members have different thicknesses the assumption is only valid if the friction at the interface between the members is sufficient to prevent slippage.

The joint geometry shown in Fig. 1 contains both an axis of symmetry and a plane of symmetry, and these can be exploited to simplify definition of the finite element model in order to considerably reduce computational expense. Axisymmetry allows the joint to be represented in cross-section rather than by a three-dimensional model; the finite element code supplies the boundary conditions necessary to impose axisymmetry while performing the solution. The symmetry of the joint about the grip midplane allows half of the joint to be removed from the model with no loss of generality; this symmetry condition is imposed upon the remaining half of the model by constraining the nodes located at the midplane to move only in the radial direction, Fig. 1(b). As Gould and Mikic demonstrated, this is not an exact representation of the behavior of a real joint because the members will separate slightly at some distance from the bolt centerline. However, analyses in which the members were allowed to separate (utilizing gap elements) resulted in stiffness values essentially the same as those in which separation was prevented. Since the latter technique entails substantially less computational expense, it was adopted for the remainder of the study.

Figure 2 shows the finite element mesh used to represent the general joint geometry of Fig. 1(b). Since only the stiffness of the members is to be considered, the shank of the bolt has been removed from the model. Further, the bolt head and the nut have also been removed, since they affect the bolt stiffness rather than that of the members. The washer is included in the model, but only as a means of applying the compressive load to the members. The elastic modulus of the washer ma-

[1]ANSYS is a registered trademark of Swanson Analysis Systems, Inc., Houston, PA.

Nomenclature

- A,B = numerical constants
- d = diameter of bolt clearance hole
- d_w = diameter of washer through which load is transferred
- E = elastic modulus of member material
- E_b = elastic modulus of bolt material
- k_1,k_2 = stiffness of individual member
- k_b = bolt stiffness
- k_m = combined stiffness of both members
- L = grip length
- r = radial coordinate
- z = axial coordinate
- α = half vertex angle of conical frustum
- ν = Poisson ratio of member material

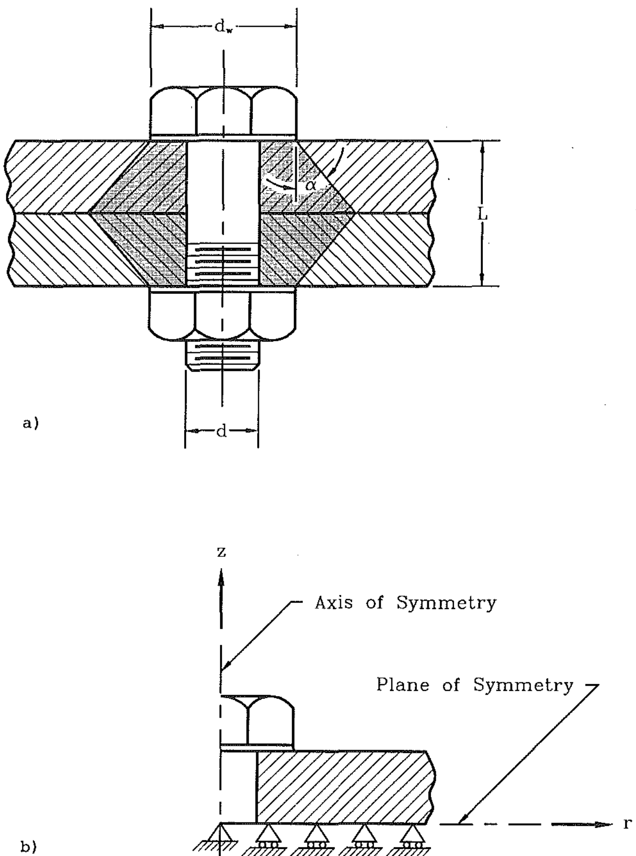

Fig. 1 (*a*) **Schematic of a typical bolted connection;** (*b*) **Connection showing symmetry assumptions used in the finite element model**

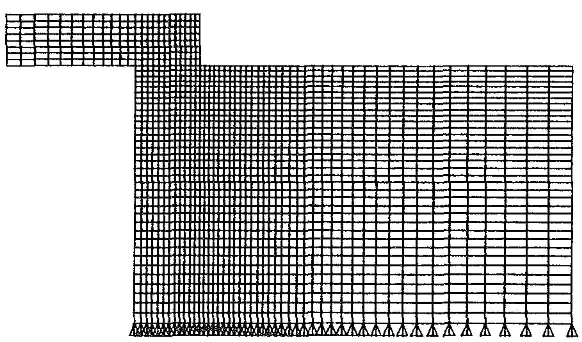

Fig. 2 The finite element model

terial is defined to be about three orders of magnitude higher than that of the members, so that the washer is essentially rigid and the deflection of the members is uniform across the interface with the washer. Although an actual washer may deform when the joint is loaded, the assumption of washer rigidity is necessary in the finite element model to prevent washer deflection from influencing the member stiffness. As stated previously, this effect can be included in the stiffness of the bolt. Because the interface between the washer and the members will be an area of stress concentration, the mesh is refined in this vicinity.

The finite element model of Fig. 2 was obtained by refining a much coarser model having elements of equal size. Each of the elements in the coarse model was divided into four elements, and the widths of these smaller elements were adjusted so that the elements are narrower in the region of stress concentration beneath the washer. Convergence of the model was verified by performing analyses for both the original coarse mesh and the mesh of Fig. 2; the results of the two analyses differed by less than two percent. The refined model of Fig. 2 contains 1976 nodes and 1872 elements and was used throughout the remainder of the analysis.

Analyses were performed for joints containing members of aluminum, cast iron, copper, and steel. In each case the material was assumed to be linearly elastic and isotropic. The compressive load was applied to the model by means of a hydrostatic pressure applied to the upper surface of the washer over an annulus between the bolt circumference and the washer circumference. In order to verify the linearity of the model, the stiffness of the members in a single model was computed for several different pressures. As expected, the stiffness was the same in each case.

To determine the effect of joint geometry upon stiffness, a number of models having different geometries were created. In each case the combined thickness of the members, L, is 25.4 mm (1.0 in.), but the hole diameters range from 2.54 to 50.8 mm (0.1 to 2 in.) in order to encompass a range of joint aspect ratios (d/L) containing most reasonable design values. For each geometry the strain energy distribution in the model was examined to detect the influence of end effects. In the case of the larger aspect ratios some end effects were noted, and the model was extended in the radial direction for these analyses to alleviate the problem.

Results

Table 1 contains the numerical results of the analyses. In each case the applied surface pressure is 17.24 MPa (2500 psi). The total load is obtained by multiplying the applied surface pressure by the annular area of the washer over which it is applied. The deflection is determined by noting the displace-

Table 1 Numerical results of the finite element analyses

		Steel		Aluminum		Copper		Cast Iron	
d/L	Force [N]	Deflection [μm]	Dimensionless Stiffness	Deflection [μm]	Dimensionless Stiffness	Deflection [μm]	Dimensionless Stiffness	Deflection [μm]	Dimensionless Stiffness
0.1	109.2	0.2647	0.7852	0.7569	0.7996	0.4562	0.7945	0.5588	0.7694
0.2	436.7	0.4796	0.8667	1.3716	0.8826	0.8230	0.8808	1.0109	0.8506
0.3	982.6	0.6655	0.9368	1.9050	0.9532	1.1430	0.9513	1.3970	0.9233
0.4	1746.8	0.8230	1.0100	2.3520	1.0293	1.4122	1.0266	1.7272	0.9957
0.5	2729.4	0.9601	1.0822	2.7584	1.0971	1.6561	1.0943	2.0168	1.0659
0.6	3930.3	1.0820	1.1523	3.1039	1.1700	1.8644	1.1665	2.2809	1.1386
0.7	5349.6	1.1735	1.2396	3.3833	1.2523	2.0168	1.2580	2.4587	1.2240
0.8	6987.3	1.2446	1.3357	3.5712	1.3558	2.1438	1.3526	2.6213	1.3122
0.9	8843.2	1.3005	1.4381	3.7135	1.4669	2.2301	1.4627	2.7432	1.4106
1.0	10917.6	1.3564	1.5321	3.8811	1.5595	2.3317	1.5544	2.8702	1.4979
1.1	13210.3	1.4021	1.6303	3.9980	1.6653	2.4028	1.6593	2.9667	1.5941
1.2	15721.3	1.4478	1.7224	4.1199	1.7629	2.4790	1.7545	3.0632	1.6843
1.3	18450.7	1.4681	1.8401	4.1808	1.8820	2.5146	1.8738	3.1242	1.7890
1.4	21398.5	1.4935	1.9479	4.2469	1.9952	2.5552	1.9858	3.1852	1.8897
1.5	24564.6	1.5189	2.0522	4.3028	2.1100	2.5908	2.0985	3.2360	1.9929
1.6	27949.0	1.5342	2.1672	4.3536	2.2244	2.6213	2.2123	3.2817	2.0962
1.7	31551.8	1.5494	2.2800	4.3840	2.3470	2.6416	2.3325	3.3172	2.2033
1.8	35373.0	1.5646	2.3906	4.4196	2.4651	2.6670	2.4462	3.3528	2.3082
1.9	39412.5	1.5799	2.4991	4.4552	2.5812	2.6873	2.5626	3.3884	2.4109
2.0	43670.3	1.5951	2.6055	4.4907	2.6956	2.7076	2.6772	3.4239	2.5114

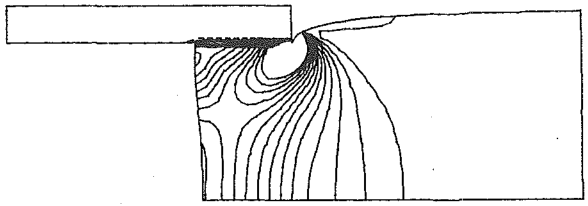

Fig. 3 Strain energy contours in a loaded connection

ment of the uppermost node located on the centerline of the washer, then doubling this value to account for the symmetric deflection of the half of the model which was removed. The dimensional stiffness is computed by simply dividing the load by the deflection.

As the ultimate goal of the study is a simple, general expression to be used in routine design problems, it is necessary to generalize the data obtained from the specific models for which finite element solutions were performed. The first procedure attempted was to refine Rötscher's method to determine an equation which would match the experimental data. The data did not closely match Eq. (3), so an attempt was made to replace the conical shape of Rötscher's stress envelope with a shape for which the integration method of Shigley and Mischke would yield stiffness values approximating those obtained from the finite element analysis.

Figure 3 contains a contour plot of the strain energy in one of the models after application of the load. The contour lines represent lines of constant strain energy. The value of the strain energy along each contour will depend upon the applied external load, but values have been omitted from the graphs as they will not affect the stiffness in a linear model. The maximum strain energy occurs at the corner of the washer as this is an area of high stress concentration, and the void beneath the edge of the washer represents these high strain energies. In an actual joint these high strain energies are frequently relieved by local yielding. The stain energy in the joint normally vanishes within three hole radii. Based upon the strain energy plot, attempts were made to approximate the stress envelope as a spherical segment and as a solid of revolution whose radial distance from the bolt centerline is described by a polynomial. None of these techniques showed any improvement over the results obtained using the conical stress envelope of Rötscher. Further, they require his assumption of uniform stress throughout planes perpendicular to the axis, which the strain energy plot of Fig. 3 shows to be invalid. This technique was ultimately abandoned, and it was decided to use the more empirical approach of simply fitting an equation to the finite element data.

Since it would be time-consuming and expensive to conduct a finite element analysis for every possible combination of material and geometry, it was desired to nondimensionalize the results of the analyses before fitting a curve to the data. Because the model is linearly elastic, the load applied to the model need not be included in the nondimensionalization process as long as the maximum stress remains within the elastic range of the material being considered. This is reasonable since the stiffness of the members should depend only upon their own intrinsic properties and not upon external loads.

The characteristic length is chosen to be the bolt diameter, d, and the elastic modulus, E, of the member material is also included in the nondimensionalization. To determine the best method of employing these parameters in the nondimensionalization, the stiffness of the bolt was first nondimensionalized. The stiffness of the bolt shank is computed in the same manner as that of a bar in simple tension:

$$k_b = \frac{AE_b}{L} = \frac{\pi E_b d^2}{4L} \tag{6}$$

where E_b represents the elastic modulus of the bolt material.

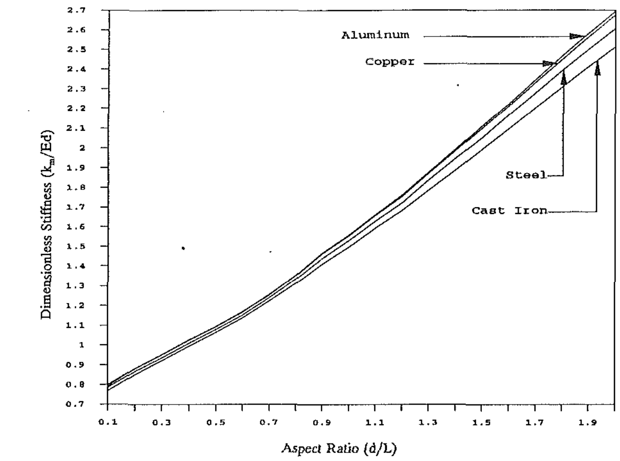

Fig. 4 Comparison of the dimensionless stiffnesses of the four materials included in the study

This expression can be reduced to a dimensionless expression in terms of joint aspect ratio by dividing both sides by $E_b d$. Thus,

$$\frac{k_b}{E_b d} = \frac{\pi}{4}\left(\frac{d}{L}\right) \tag{7}$$

Here, the dimensionless stiffness of the bolt is a function of aspect ratio, d/L. Therefore, a reasonable approach is to derive an expression for the member stiffness in which k_m is also a function of the aspect ratio, d/L; i.e.,

$$\frac{k_m}{Ed} = f(d/L) \tag{8}$$

Values of the dimensionless stiffness, k_m/Ed, are provided in Table 1 as a function of the joint aspect ratio, d/L, and these values are represented graphically in Fig. 4. It is desired to find an expression having the form of Eq. (8) which closely matches the data obtained from the finite element analysis without adding too many additional parameters to the problem. Two types of equations were tested. A polynomial was first used to approximate the finite element results, but could not be made to match the data satisfactorily. Adding terms to the polynomial increased the accuracy only marginally while greatly increasing the complexity of using the equation in design problems. For this reason the idea of using a polynomial was abandoned.

The next functional relationship chosen was an exponential expression of the form

$$\frac{k_m}{Ed} = Ae^{B(d/L)} \tag{9}$$

Using a least squares routine to fit an equation of this form to each set of finite element data resulted in a curve which passed almost directly over each data point.

Although the dimensional stiffnesses computed for models having the same geometry varied greatly from one material to another, the dimensionless stiffness varied only slightly, as evidenced by Fig. 4. This slight variation results from the dependence of the stiffness upon the Poisson ratio of the material. Several attempts were made to incorporate the Poisson ratio into the nondimensionalization process, but no simple method of including ν provided a close match with the finite element results. However, an investigation of the effects of Poisson's ratio upon the dimensionless stiffness showed that within the range between $\nu = 0.2$ and $\nu = 0.35$, which contains most engineering materials, the effect on the stiffness is minimal (Choudhury, 1988).

Table 2 shows the coefficients A and B which result from fitting Eq. (9) to the finite element data for the four materials

Table 2 Stiffness parameters for engineering materials

Material Used	Poisson Ratio	Elastic Modulus [GPa]	A	B
Steel	0.291	206.8	0.78715	0.62873
Aluminum	0.334	71.0	0.79670	0.63816
Copper	0.326	118.6	0.79568	0.63553
Gray Cast Iron	0.211	100.0	0.77871	0.61616
General Expression			0.78952	0.62914

included in the study. To compute the stiffness of members made from a different material, the values of A and B corresponding to the material having the closest Poisson ratio to the material of the actual joint should be chosen. Since the effect of Poisson's ratio upon dimensionless stiffness is very small, values for A and B obtained by fitting an equation to the data for all four materials are also included in the table. These values will result in stiffness values slightly different from those obtained by using the exact A and B for the material of the joint. However, they allow the use of a single expression which provides a reasonable approximation for the stiffness of members made of any engineering material.

As a numerical example, we shall compute the stiffness for members having the geometry of the experimental apparatus of Maruyama et al. (1975) so that the resulting stiffness can be compared to that determined from their experimental work. They investigated a joint consisting of steel members having a hole diameter of 25 mm and a grip length of 50 mm. They performed experiments for cylindrical members having outer diameters of several values, but only their experimental results for members having an outer diameter of 100 mm approach the requirement that the member radius be at least 3 times the hole radius to avoid end effects. Their experiments with this geometry yielded member stiffnesses of 5.11×10^9 N/m (521 kgf/μm) and 5.54×10^9 N/m (565 kgf/μm) for two different grades of steel. To perform the stiffness calculation using Eq. (9), note that the aspect ratio, d/L, is 0.5, and the elastic modulus of steel is 206.8 GPa. Thus the stiffness is computed as

$$k_m = EdAe^{B(d/L)}$$
$$= (206.8\times10^9 \text{ N/m}^2)(0.025 \text{ m})(0.78715)e^{0.62873(0.5)}$$
$$= 5.57\times10^9 \text{ N/m } (568 \text{ kgf}/\mu\text{m}) \quad (10)$$

This stiffness matches the experimental results stated above more closely than Maruyama's own prediction of 6.29×10^9 N/m (641 kgf/μm).

Discussion

The stiffness of the members for a particular joint can be computed by simply substituting the appropriate numerical values into Eq. (9) and solving for k_m. However, it is important to appreciate the assumptions inherent in this expression when using it to design an actual joint. In particular, when deciding upon a safety factor, it is important to consider the degree to which the joint being designed satisfies the constraints of the finite element analysis. If the distance from the bolt axis to the edge of the members is not several times the bolt diameter, end effects must be considered. Excessive thread friction, shear loads, slippage at the member interface, rough surface finish, and other violations of the assumptions may affect the accuracy of the stiffness computation, and safety factors should be applied accordingly if any of these effects is present. Equation (9) should never be applied to a joint containing an unconfined gasket. In joints subjected to cyclic loading, the effects of yielding or crack growth may be pronounced at stress concentrations, and safety factors should be increased accordingly.

If a joint contains members made of different materials, it is no longer symmetric about its midplane, and Eq. (9) is likely to be in error. However, members of different materials can be accommodated by altering the procedure slightly if the members are of the same thickness. Since the two members act as springs arranged in series, the stiffness of each member can be computed separately, then the two can be combined using

$$\frac{1}{k_m}=\frac{1}{k_1}+\frac{1}{k_2} \quad (11)$$

where k_1 is the individual stiffness of the first member and k_2 is the individual stiffness of the second member. The stiffness of an individual member is obtained by computing the stiffness of the entire joint as if it were made of the material of the single member, then multiplying by two. Additional finite element analyses (Choudhury, 1988) have shown that the error resulting from this technique is slight.

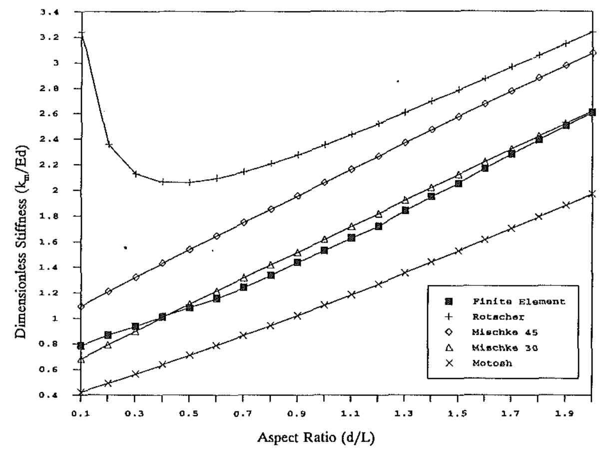

Fig. 5 Comparison of Eq. (9) to current analysis techniques

Figure 5 shows a comparison of stiffnesses computed using Eq. (9) with those resulting from several of the techniques commonly used. It is interesting to note that with the exception of Motosh's work, all of the currently used design techniques overestimate the stiffness of the members and will, therefore, underestimate the portion of an external load which must be supported by the bolt. In a bolted joint designed for fatigue resistance, for example, this will result in an underestimate of the alternating stresses in the bolt, creating the possibility of bolt failure. The member stiffness will also affect the sealing effectiveness of a joint with metal-to-metal contact or with a confined gasket in that the relative stiffnesses of the bolt and members determine the external load required to induce separation in the connection. In this case, overestimating the stiffness of the members imposes an unnecessarily stringent condition upon the preload which must be applied to the bolt.

It is also interesting to note that the result of Shigley and Mischke using $\alpha=30$ deg, although not in widespread use, agrees very closely with the finite element results. It is believed, however, that Eq. (9) is more convenient to use, and it does not present the conceptual difficulty of assuming layers of constant stress and a discontinuity in the stress distribution on the boundary of an arbitrary envelope.

Conclusions

Finite element analyses have been performed for models of bolted joints having a range of geometries and materials. A method of nondimensionalization is suggested, and an exponential expression relating the dimensionless stiffness to the joint aspect ratio and material elastic modulus has been found to closely match the finite element data. A small dependence upon Poisson's ratio has been noted and accounted for by two numerical constants which depend upon the material of the joint. Comparisons have been made with techniques currently used to compute member stiffness, and most are found to overestimate the magnitude of the stiffness.

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