VOLUME 34 (1991), 2, 169–176
TRIBOLOGY TRANSACTIONS

Dynamic Response to Axial Oscillation and Rotating Seat Runout in Contacting Mechanical Face Seals©

ITZHAK GREEN (Member, STLE) and SCOTT BAIR
The George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0405

Contacting mechanical face seals are ubiquitous in many industries, e.g., petrochemical seals, automotive water pump seals, and low pressure gas seals. Environmental awareness and new strict regulations may force an even wider use of contacting seals in limiting leakage of toxic fluids. Contacting operation is achieved when the closing force exceeds a presumed opening force. For prolonged seal life it is desired that the contacting force be as small as possible. On the other hand, the contacting force must be big enough to ensure face contact. To maintain contacting operation the shaft speed must be smaller than some critical value, known as the separation speed. The separation speed is affected by many dynamic parameters of the system, among which is the damping of the support. This damping is either inherent in the system, in the form of a secondary seal, or is introduced especially to limit, for example, metal bellows radial vibration. The analysis herein complements an earlier investigation which was limited to undamped seals only. The current analysis reveals that support damping has an adverse effect on the separation speed and wear. Hence, an undamped seal is preferable for a flexibly mounted stator contacting seal configuration.

Presented at the 45th Annual Meeting
In Denver, Colorado
May 7–10, 1990
Final manuscript approved January 5, 1990

INTRODUCTION

Major advances in knowledge and technology of noncontacting mechanical face seals have been reported (*1*), (*2*). The majority of seals in many industries, however, are still of the contacting type (or at least they have been designed to be such). For example, Nosaka et. al. (*3*) report an analytical and experimental study to develop a high speed contacting mechanical seal for liquid hydrogen turbopump of a liquid oxygen and liquid hydrogen rocket engine.

The advantage of contacting seals over noncontacting seals is that leakage is practically zero in the former case. Contacting between seal faces is maintained if the closing force on the flexibly mounted element, always exceeds any open-

NOMENCLATURE

D = axial support damping
d = angular support damping
F = force
F_p = a component of the total force caused by a non-uniform pressure distribution
F_{total} = total load on the stator resulting from the interface
F_ζ = compression force in axial mode
I = stator transverse moment of inertia
K = axial support stiffness
k = angular support stiffness
M = moment
m = stator mass
R = radial location at which the stator and rotor make contact
r = frequency ratio
r_s = radial location of flexible support
t = time
Z = stator axial degree of freedom
γ_r = rotor misalignment (runout)
γ_s = stator tilt (nutation)
ΔZ = flexible support axial preset
δ = radial location of the center of pressure
η = damping parameter
ζ = axial pulsation of the shaft
ζ_o = amplitude of axial pulsation of the shaft
ψ = precession
ω = shaft angular velocity
ω_n = natural frequency (axial or angular)
ω_{sep} = shaft angular velocity to cause separation onset
ω_ζ = frequency of axial pulsation of the shaft

SUBSCRIPTS

e = secondary seal
ex = after relaxation
i = initial (but after assembly and relaxation)
sp = restoring element
t = time component

ing forces that might be generated in the sealing dam due to fluid film effects, or inertia (dynamic/vibration) effects. As contact is the mode of operation, wear of the seal faces is inevitable. Hence, the seal has to be carefully designed such that the closing force should be big enough to guarantee contact under varying operating conditions, but small enough to minimize wear and to prolong seal life.

One mode of oscillation, in the form of chatter, may occur at low rotational speed due to the stick-slip effect. At moderate and high rotational speeds the inertia effect becomes more pronounced and may cause face separation. The speed at which the seal faces would separate was the subject of an investigation by Zorowski et. al. in a series of papers (4)–(6). A major assumption used throughout their work was that the shape of the normal force distribution, generated by wobble and axial pulsation, is assumed to be linear with respect to a diameter of the ring. The equations of motion of the rigid ring were used to determine an expression for the magnitude and distribution of the assumed contact force. This equation was minimized with respect to time and position on the ring and set equal to zero. This (according to Zorowski et. al.) corresponded to mathematically applying the physical criterion which defines the onset of mechanical separation. Justification for the assumed nature of the contact force was not provided.

In a recent study (7) a different contact mechanics model was suggested. It was demonstrated that even if the minimum contacting force equals zero (and it can be zero all over the interface but one point) full contact can still be maintained. This finding is in disagreement with the major assumption in prior works. The alternate contact mechanics model leads, of course, to a separation speed which is different from that offered by Zorowski et. al. The previous study (7), however, was limited to an undamped seal. An undamped model can rarely simulate a real seal. Structural support damping (which mostly originates at the secondary seal) is inevitable and must be considered in any realistic design. This work supplements (7) to include support damping effects on the separation speed in axial and angular modes. The contacting model and separation criterion which was laid out in the previous study will be used here as well.

CONFIGURATION AND ASSEMBLY CONSIDERATION

The number of actual seal configurations and assemblies is too great for all to be considered in a single analysis. However, two basic configurations can be identified based on the flexible support type. The first type is a metal bellows seal (see for example (7)), and the second type is a seal with circumferential springs and a secondary seal, usually an elastomeric O-ring, as shown schematically in Fig. 1. During assembly the two faces are pressed together to form a tight sealing dam for contacting operation. The amount of load that remains in the support after assembly depends upon the support type. The support may be purely elastic (as in the case of metal bellows), or it may contain some hysteresis, undergo frictional sliding and experience relaxation (as in the case of an elastomeric secondary seal).

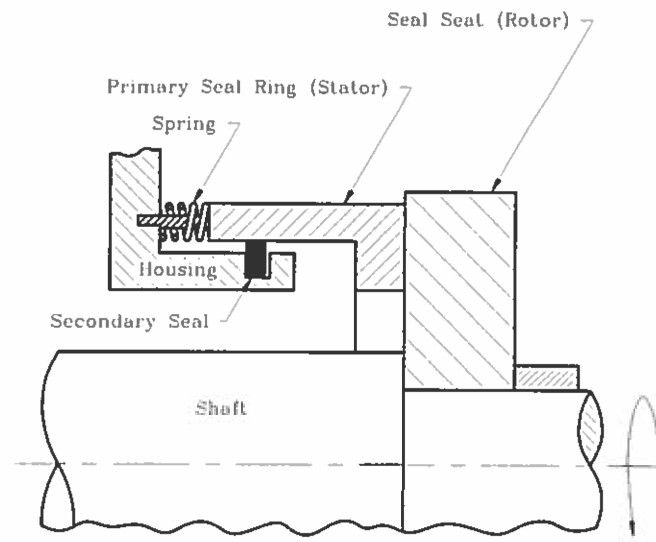

Fig. 1—Schematic of a contacting mechanical seal

Relaxation Test

A simple experiment was performed on a commercial O-ring (Buna-N, 50 mm nominal diameter, and 2.62 mm wire diameter) to investigate the extent to which the axial installation force relaxes with time. The test apparatus is shown in Fig. 2. A press drives a shaft axially inside a cylinder fitted with an internal groove. The O-ring was installed with nine percent squeeze.

The results in Fig. 3 were obtained by rapidly driving the shaft through the cylinder (about 1 mm displacement) until sliding was clearly sensed (the reading from the load cell reached its maximum) and stopping by locking the press position. The force in the load cell versus time shows the relaxation behavior of the O-ring for which a mechanical equivalent is also shown in Fig. 3. It can be seen that the force drops quite rapidly (about 50 percent in 10 seconds) and levels off to about 40 percent of the maximum initial force. That force will be referred to as F_{ex}.

Force Analysis

Let F_a be the contact force at the interface (between the seal faces) at the assembly time due to a compression displacement. For a purely elastic support F_a is entirely a restoring force. For a support that contains springs and an elastomeric secondary seal, the force F_a is composed of a restoring force in the springs, F_{sp}, and a force in the elastomer, F_e. Hence,

$$F_a = F_{sp} + F_e$$

In the latter support case, some time after assembly, the force F_e relaxes (as described above) and the force that remains is $F_{sp} + F_{ex}$, where F_{ex} is the force in the elastomeric seal after relaxation. Hence, if F_i is the force acting on the stator at the interface at rest (after assembly and relaxation), one gets

$$F_i = F_{sp} + F_{ex} \qquad [1a]$$

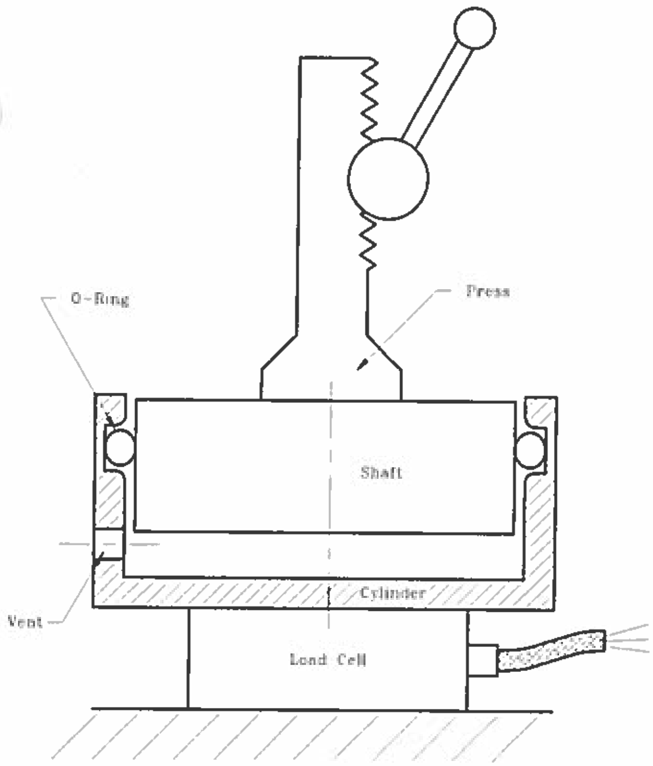

Fig. 2—Schematic of O-ring axial force relaxation measurement

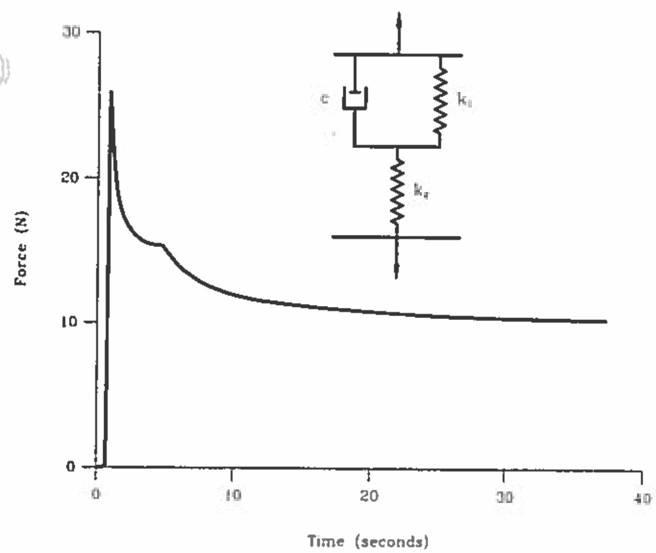

Fig. 3—Force relaxation and a mechanical equivalent

The separation criterion depends greatly on that initial force (7). The higher that force is the higher the separation speed (and wear). (It may be unsafe to consider F_{ex} as a restoring force. During operation further relaxation may occur and actually F_{ex} may diminish altogether. Hence, the only compression force that is safe to be considered is the force that can be restored from a purely elastic element such as metal bellows or springs.) Designating K_{sp} as the total axial stiffness coefficient of the restoring element(s), yields

$$F_{sp} = K_{sp}\,\Delta Z' \qquad [1b]$$

where $\Delta Z'$ is the preset of the restoring element(s) only.

ANALYSIS

Axial Mode

The mechanical seal of Fig. 1 is modeled in Fig. 4(a) as a lumped parameter system. The mass of the stator is m, the axial support damping coefficient is D, and the total axial support stiffness is

$$K = K_{sp} + K_r \qquad (2)$$

where K_r is the secondary seal dynamic axial stiffness coefficient. (The values of the dynamic stiffness and damping coefficients of the secondary seal, K_r and $D_r \equiv D$, respectively, need normally to be determined from experimental work, e.g., see Ref. (8). The reason for considering a total stiffness is that once the seal is assembled and placed in service, the secondary seal becomes effective in dynamically resisting small oscillation about an equilibrium position. The displacements are small enough to assume that slippage does not occur and that a Kelvin-Voigt model can be applied (8). For mathematical treatment it is convenient to define an effective support preset, ΔZ, such that

$$\Delta Z = F_i/K \qquad (3)$$

where F_i is given in Eq. [1].

Axial oscillation of the mating ring (rotor) is simulated by

$$\zeta = \zeta_o \sin\omega_\zeta t \qquad [4]$$

where ζ_o and ω_ζ are the amplitude and angular frequency of the forced oscillation, respectively. For the free body diagram in Fig. 4(b), the following equation of motion applies:

$$m\ddot{Z} = -K(Z + \Delta Z) - D\dot{Z} + F_\zeta \qquad [5]$$

where F_ζ represents the contacting force acting on the stator at the interface. (Note that at rest $Z = \dot{Z} = \ddot{Z} = 0$, and $F_\zeta = K\Delta Z = F_i$.) The kinematical condition for contacting operation is $Z = \zeta$. Substituting Eq. [4] in Eq. [5] and solving for F_ζ gives

$$F_\zeta = K\Delta Z + F_t \qquad [6a]$$

at

$$F_t = \zeta_o \sin\omega_\zeta t\,(K - m\omega_\zeta^2) + \zeta_o\,\omega_\zeta D\cos\omega_\zeta t \qquad [6b]$$

where F_t represent the time dependent force component of F_ζ. The force F_ζ and its various components are shown schematically in Fig. 5. As long as $F_\zeta > 0$ contacting operation prevails. Separation onset occurs at $F_\zeta = 0$, or

$$K\Delta Z + F_t|_{\min} = 0 \qquad [6c]$$

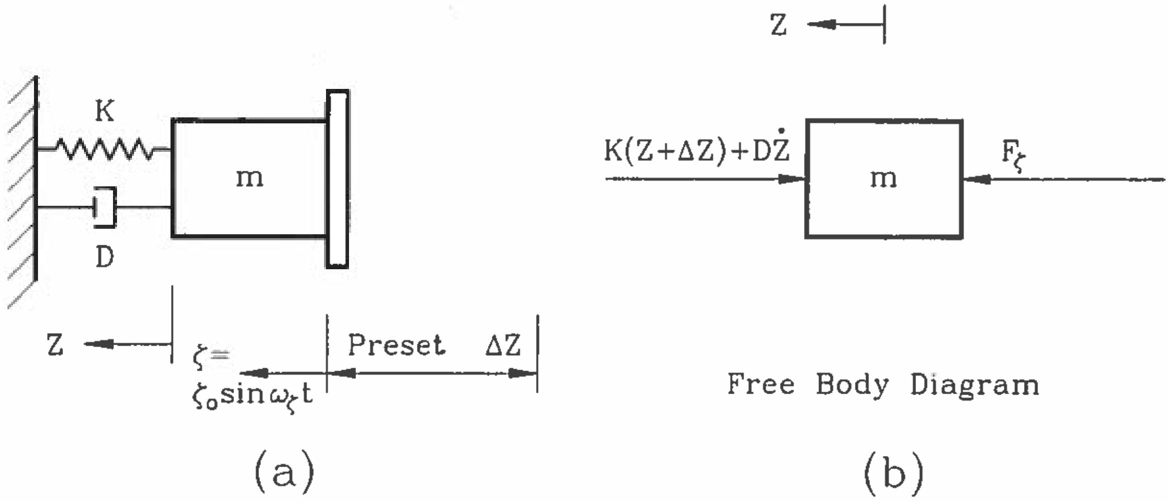

Fig. 4—Lumped parameter model for the axial pulsation mode

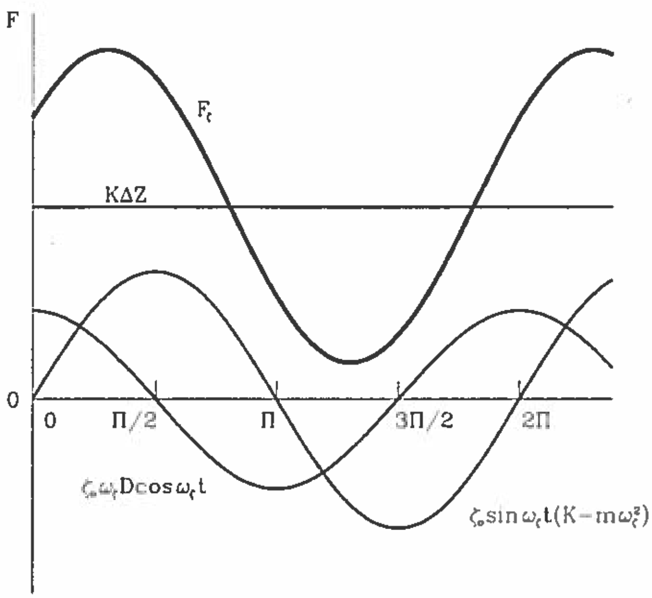

Fig. 5—Force components and the total contacting force as a function of the circular shaft location.

The derivative of F_t in Eq. [6b] is taken with respect to time and set to zero. This yields the circular position at which F_t is minimum:

$$\tan \omega_\zeta t \,|_{\min} = \frac{K - m\omega_\zeta^2}{\omega_\zeta D} = \frac{1 - r^2}{2\eta r} \qquad [7a]$$

where

$$\omega_n^2 = \frac{K}{m}\,; \qquad \eta = \frac{D}{2m\omega_n}\,; \qquad r = \frac{\omega_\zeta}{\omega_n} \qquad [7b]$$

The newly defined variables ω_n, η, and r are the natural frequency, damping parameter, and frequency ratio, respectively. Note that the solution of Eq. [7a] results in a circular position in regions $\pi \leq \omega_\zeta t|_{\min} < 3\pi/2$ for $K - m\omega_\zeta^2 \geq 0$, and $\pi/2 < \omega_\zeta t|_{\min} \leq \pi$ for $K - m\omega_\zeta^2 \leq 0$. These regions were observed graphically in plots such as in Fig. 5.

Substituting the result [7a] in Eq. [6b] and then in Eq. [6c], yields the criterion for separation onset in the axial mode,

$$\frac{\Delta Z}{\zeta_o} = [(1 - r^2)^2 + (2\eta r)^2]^{1/2} \qquad [8]$$

It is interesting to note that for an undamped seal, $\eta = 0$, Eq. [8] yields the same separation criterion found in Ref. (7), Eq. [4]. Equation [8] is shown graphically in Fig. 6 for various levels of damping. Since wear is directly proportional to face loading, F_t, and by Eq. [3] to ΔZ, it is desirable to have ΔZ as small as possible. It is clearly seen in Fig. 6 that damping has a detrimental effect on the separation condition since it reduces the region at which contacting operation prevails. The higher the damping the higher ΔZ is required for assuring contacting operation. Hence, an undamped seal is preferable.

From Fig. 6 it can be seen that there is a minimum point which a design should be aimed at. That minimum is found by setting the derivative of Eq. [8], with respect to r, to zero. This yields two extremums; one at $r = 0$, and the second at

$$r = \sqrt{1 - 2\eta^2} \quad @ \quad \eta < \sqrt{1/2}$$

The undamped seal, $\eta = 0$, is shown to require zero preset at resonance ($r = 1$) and is, therefore, the best seal operating at the best design condition.

There is, however, a small technical problem in reaching the optimum point at start-up (or going through shut-down), since the seal would experience noncontacting operation in the transient. To eliminate this problem a temporary additional preload would be necessary to suppress separation. Once the seal has reached its operating speed the temporary load can be removed (in order to minimize wear). Also, from a practical viewpoint, it is recommended to provide a preset slightly above the optimal value. This will allow for some

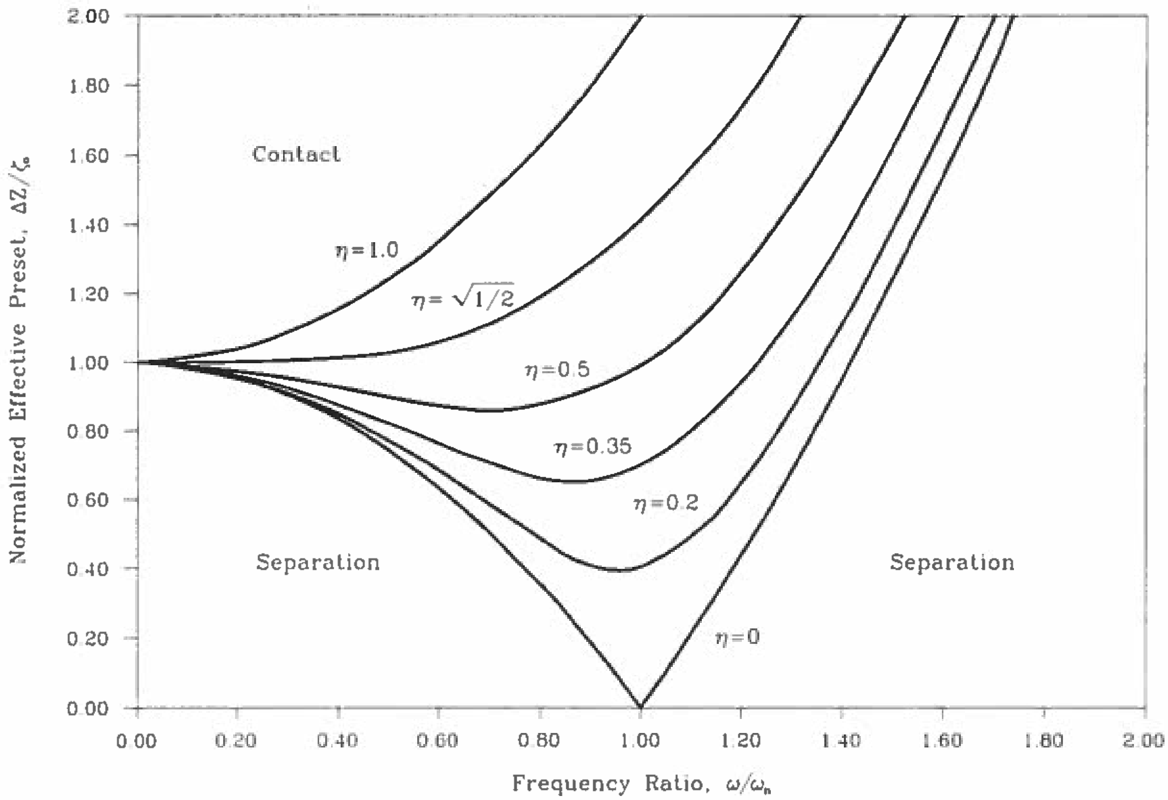

Fig. 6—Separation onset map in the axial mode

variations (disturbances) in the shaft speed and still remain in the contacting operation region. The results of Fig. 6 also indicate that seals should always have a natural frequency larger than the operating speed ($r < 1$); otherwise, the required preset grows quite rapidly with r when $r > 1$.

Angular Mode

The mechanics of contact was presented in detail in (7). The final stage of face attachment is shown in Fig. 7. At first, a displacement Z_c is needed to bring the stator into full contact with the rotor such that the stator assumes the rotor runout, γ_r. Subsequently an additional preset, ΔZ, is applied in order, for example, to prevent separation due to axial mode vibrations (as explained previously). Again, Z_c and ΔZ are effective compressions based on restoring portions of the force at attachment. The total force due to Z_c and ΔZ is given by

$$F_{total} = K\,(Z_c + \Delta Z)$$

In Ref. (7) it was found that $Z_c = R\gamma_r/2$ where R is the radius at which contact is made. The criterion for separation is also repeated from that work. That is, for the faces to separate the force that acts on the stator, due to dynamic effects, must be bigger than the force that was used to attach the two elements. In other words, the dynamic force must overcome the internal force stored in the restoring elements to cause the faces to pull apart. At separation onset the stator is about to pitch at some point, p, as shown in Fig. 7. At separation onset the contact force is wholly concentrated at point p, and it equals the total force. Hence,

$$F_p\Big|_{\text{separation onset}} = K\left(\frac{R\gamma_r}{2} + \Delta Z\right) \qquad [9]$$

The stator angular equations of motion (which have been derived elsewhere (9) are

$$I\,(\ddot{\gamma}_s + \dot{\psi}^2\gamma_s) = M_x \qquad [10a]$$

$$I\,(\ddot{\psi}\gamma_s + 2\dot{\gamma}_s\,\dot{\psi}) = M_y \qquad [10b]$$

The moments M_x and M_y are the applied moments that act upon the stator as shown in Fig. 7, where I is the transverse moment of inertia. Axis x designates the axis of stator nutation and axis y is perpendicular to x in the stator plane. The two axes precess at a rate $\dot{\psi}$. While contact is maintained and the stator tracks the misaligned wobbling rotor at a rate ω (being the shaft angular velocity), the following kinematical conditions apply:

$$\dot{\psi} = \omega = \text{const};\ \ddot{\psi} = 0 \qquad [11a]$$

$$\gamma_s = \gamma_r = \text{const};\ \ddot{\gamma}_s = \dot{\gamma}_s = 0 \qquad [11b]$$

The applied moments contributed by the flexible support were derived in Eqs. [54] and [55] in Ref. (10) (see also

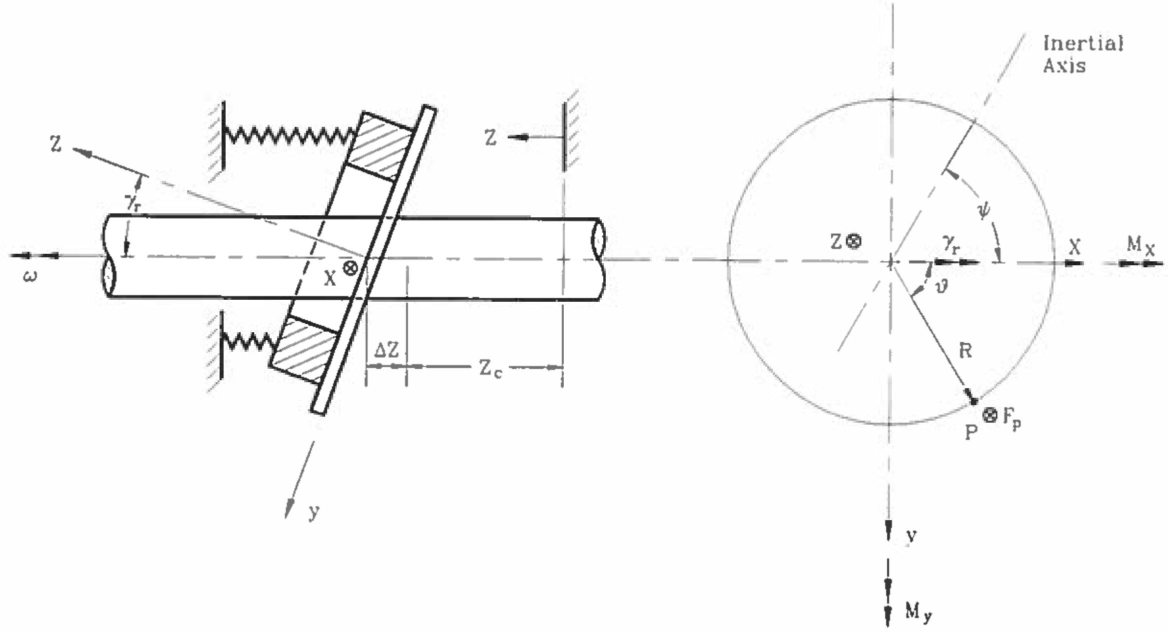

Fig. 7—Final stage of attachment, coordinate system, tilt, and applied moments

definitions of angular stiffness and the damping coefficient) to be

$$M_x = -k\,\gamma_s - d\dot{\gamma}_s$$

$$M_y = -d\gamma_s\dot{\psi}$$

where according to Ref. (*10*) the angular stiffness and damping coefficients are obtained from their corresponding axial values by:

$$k = 1/2\,K\,r_s^2$$

$$d = 1/2\,D\,r_s^2$$

r_s being the radial location of the flexible support.

Hence, with these equations and the conditions in Eqs. [11] the equations of motion [10] at separation onset, $\omega = \omega_{sep}$, reduce to

$$-I\,\omega_{sep}^2\,\gamma_r = -\,k\,\gamma_r + F_p\,(R\sin\theta) \qquad [12a]$$

$$0 = -d\,\gamma_r\omega_{sep} - F_p\,(R\cos\theta) \qquad [12b]$$

The last terms on the right hand side of Eqs. [12] represent the moments that the force F_p at the contact point, p, contributes to the applied moments on the stator. For mathematical convenience, and conciseness of the derivation, r_s will be replaced by R. First, this is a fairly reasonable assumption for many practical seals. Second, it will be easier to interpret final results without the complexity of the presence of two geometrical quantities of the same nature. Hence,

$$k = 1/2\,K\,R^2 \qquad [13a]$$

$$d = 1/2\,D\,R^2 \qquad [13b]$$

Also, for a narrow annular stator the transverse moment of inertia is assumed to be

$$I = 1/2\,mR^2 \qquad [13c]$$

It is interesting to note that for an undamped seal, $d = 0$, the only solution for Eq. [12b] is $\theta = \pi/2$, $3\pi/2$. The first solution, $\theta = \pi/2$, represents the case when the contacting point is first made on axis y (see Fig. 7) at rest, $\omega = 0$. The second solution, $\theta = 3\pi/2$, represent the case of separation onset, $\omega = \omega_{sep}$, to indicate that the pitch point, p, has moved 180° from static attachment. This totally agrees with the previous analysis in Ref. [7].

Elimination of the trigonometric functions in Eqs. [12] is done by adding the squares of the equation. Hence,

$$(k - I\,\omega_{sep}^2)^2 + d^2\,\omega_{sep}^2 - \left(\frac{F_pR}{\gamma_r}\right)^2 = 0 \qquad [14]$$

Of a small practical significance is the angular position at which separation onset occurs. At any rate, from Eq. [12] one gets

$$\tan\theta = \frac{I\omega_{sep}^2 - k}{d\,\omega_{sep}}$$

Substituting Eq. [9] in [14] and solving for the separation speed squared, yields

$$\omega_{sep}^2 = \frac{k}{I} - \frac{d^2}{2I^2} \pm \sqrt{\frac{d^4}{4I^4} - \frac{d^2}{I^2}\frac{k}{I} + \left(\frac{k}{I}\right)^2\left(1 + \frac{2\Delta Z}{R\gamma_r}\right)^2}$$

Definitions similar to Eq. [7b] are provided for the angular mode as follows:

$$\omega_n^2 = \frac{k}{I} \; ; \qquad \eta = \frac{d}{2I\omega_n} \qquad [15]$$

Hence,

$$\left(\frac{\omega_{sep}}{\omega_n}\right)^2 = 1 - 2\eta^2 \pm \sqrt{4\eta^4 - 4\eta^2 + \left(1 + \frac{2\Delta Z}{R\gamma_r}\right)^2} \qquad [16]$$

It is of interest to examine again the undamped case, $\eta = 0$. Only the positive sign yields a physical solution (which is identical to Eq. [31] in Ref. (7). But this is not the only reason for discarding the negative sign from Eq. [16]. The worst case that could possibly make the discriminant negative is when $\Delta Z = 0$. For which case, a simple investigation reveals that the discriminant is zero only when

$$\eta = \sqrt{1/2}$$

and positive for any other η. Hence, for $\Delta Z \geq 0$ the discriminant is greater than or equal to zero, and the square-root yields a real value. However, the higher ΔZ is, the higher that real value is, independently of η. Physically, the higher ΔZ is, the higher the separation speed is; hence, the positive sign is appropriate. Otherwise, the negative sign would yield an imaginary separation speed which is not physically feasible.

An understanding of the damping effect can be obtained by examining again the case of $\Delta Z = 0$. Equation [16] results in

$$\frac{\omega_{sep}}{\omega_n} = (1 - 2\eta^2 + |\,2\eta^2 - 1\,|)^{1/2} \qquad [17]$$

The highest separation speed is obtained when $\eta = 0$ resulting in

$$\omega_{sep} = \sqrt{2}\,\omega_n$$

(which agrees with Eq. [27] in Ref. (7)). As η increases ω_{sep} decreases, and for

$$\eta \geq \sqrt{1/2}$$

Equation [17] results in $\omega_{sep} = 0$ which indicates imminent separation right at start-up. Hence, it is concluded that damping has a negative effect on the separation speed.

The general solution for the separation speed (including the preset ΔZ) is obtained from Eq. [16] to be

$$\frac{\omega_{sep}}{\omega_n} = \left[1 - 2\eta^2 + \sqrt{4\eta^4 - 4\eta^2 + \left(1 + \frac{2\Delta Z}{R\gamma_r}\right)^2}\,\right]^{1/2} \qquad [18]$$

The normalized separation speed, ω_{sep}/ω_n, in Eq. [18] is shown graphically in Fig. 8 as a function of the normalized preset, $\Delta Z/R\gamma_r$, at different levels of damping. It is clearly seen, once again, that damping has a detrimental effect on the separation speed. As damping parameter,

$$\eta \geq \sqrt{1/2}$$

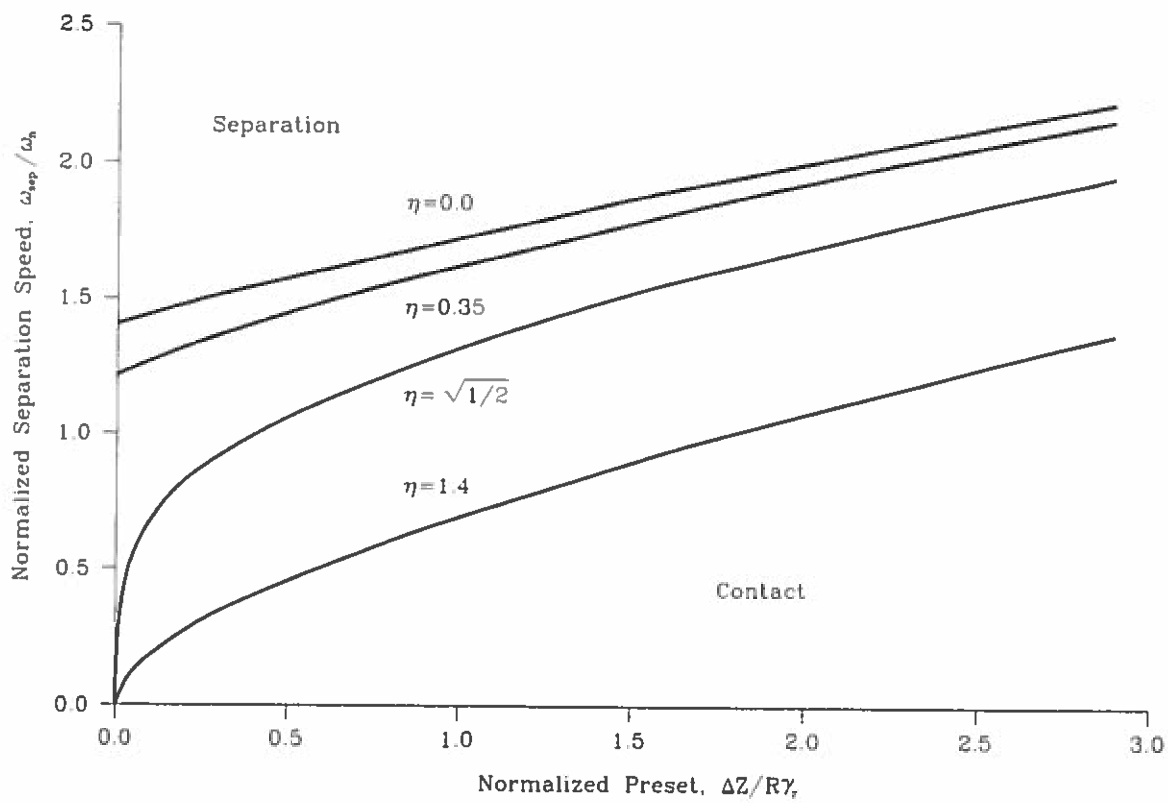

Fig. 8—Separation onset map in the angular mode

results in $\omega_{sep} = 0$ when $\Delta Z = 0$. In other words, a seal with

$$\eta \geq \sqrt{1/2}$$

demands a preset, which sometimes can be substantial, in order to prevent separation even at low shaft speeds.

The force F_p in Eq. [14] was shown in Ref. (7) to represent a force caused by the nonuniformity of the contact pressure. This force, being a concentrated force, was associated with the wear of the faces since it acts like a pin-on-disk tester, (the pin is rubbing against a rotating disk). Therefore, it is desirable to minimize F_p. To achieve this goal, Eqs. [12] are repeated for a general shaft speed, ω, such that $\omega \leq \omega_{sep}$

$$-I\,\omega^2\gamma_r = -k\gamma_r + F_p\,(\delta\sin\theta)$$

$$0 = -d\gamma_r\omega - F_p\,(\delta\cos\theta)$$

where δ is the radial location of the center of pressure (7). The center of pressure is the location at which F_p (the force equivalent to the non-uniform contact pressure) acts. Using the definition in Eq. [15] and repeating the stages that lead to Eq. [14], results in

$$\frac{F_p\delta}{\gamma_r k} = [(1 - r^2)^2 + (2\eta r)^2]^{1/2} \qquad [19]$$

where $r = \omega/\omega_n$. Equations [19] and [8] are identical in form, and the ordinate in Fig. 6 can represent the normalized moment $F_p\delta/\gamma_r k$. The minimization procedure that was performed on Eq. [8] applies to Eq. [19]. Namely, there are two extremum points, being minimum or stationary points, depending on the following two regions of η:

$$\text{I. } 0 \leq \eta < \sqrt{1/2} \quad \begin{cases} r = 0 \text{ is a stationary point} \\ r = \sqrt{1 - 2\eta^2} \text{ is a minimum point} \end{cases}$$

$$\text{II. } \eta \geq \sqrt{1/2} \qquad r = 0 \text{ is a minimum point.}$$

The undamped seal, $\eta = 0$, has its minimum at resonance ($r = 1$), and by Eq. [19] $F_p = 0$. Hence, the undamped seal proves again to be the best seal in minimizing wear, especially when it operates at resonance. This result complies with the finding in (7).

CONCLUSIONS

A closed form analytical solution to the problem of the separation speed of contacting mechanical face seals has been presented. This work especially concentrated on the effects of support damping. It was found that support damping adversely affects the separation speed and wear, both in axial and angular modes. An optimum operation speed was found (in each mode) in which wear (and, therefore, the contact force necessary to prevent separation) would be minimum. That speed is

$$\left(\frac{\omega}{\omega_n}\right)_{\text{optimum}} = \sqrt{1 - 2\eta^2} \text{ for } \eta \leq \sqrt{1/2}$$

Values of $\eta > \sqrt{1/2}$ result in presets that are quite high and should be avoided. It should be noted that the above optimum speed is always less than the separation speed, given by Eq. [18]. Hence, the optimum speed is a safe speed.

REFERENCES

(1) Allaire, P. E., "Noncontacting Face Seals for Nuclear Applications—A Literature Review," *Lubr. Eng.*, **40**, 6, pp 334–351, (1984).

(2) Etsion, I., "Mechanical Face Seal Dynamics Update," *The Shock and Vibration Digest*, **17**, 4, pp 9–14 (1985).

(3) Nosaka, M., et. al., "Sealing Characteristics of High Speed Face-Contact Metal Bellows Mechanical Seals for Liquid Hydrogen," *Proc. of JSLE Intl. Trib. Conf.*, Elsevier Science Publ. Co., New York, pp 823–828 (1985).

(4) Hart, F. D. and Zorowski, C. F., "Onset of Mechanical Separation in Bellows Supported Rotary Face Seals," ASME 1965 Gas Turbine Conf. Paper **65-GTP-4.**

(5) Hart, F. D. and Zorowski, C. F., "Dynamic Coupling Effects in Rotary Face Seal Separation Phenomena," *Trans. of ASME, Jour. of Eng. for Industry*, **89**, 2, pp 296–300 (1967).

(6) Zorowski, C. F. and Hill, H. H., "Post Mechanical Separation in Elastically Supported Rotary Face Seals," *ASLE Trans.*, **14**, 1, pp 75–80 (1971).

(7) Green, I., "Separation Speed of Undamped Metal Bellows Contacting Mechanical Face Seals," *STLE Trib. Trans.*, **33**, 2 (1990).

(8) Green, I. and Etsion, I., "Pressure and Squeeze Effects on the Dynamic Characteristics of Elastomer O-rings Under Small Reciprocating Motion," *Trans. ASME, Jour. of Trib.*, **108**, 3, pp 439–445 (1986).

(9) Green, I. and Etsion, I., "A Kinematic Model for Mechanical Seals with Antirotation Locks or Positive Drive Devices," *Trans. ASME, Jour. of Trib.*, **108**, 1, pp 42–45 (1986).

(10) Green, I. and Etsion, I., "Stability Threshold and Steady-State Response of Noncontacting Coned Face Seals," *ASLE Trans.*, **28**, 4, pp 449–460 (1985).