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Exact Spectral Moments and Differentiability of the Weierstrass-Mandelbrot Fractal Function

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The Mathematica code can be obtained from the author upon request.

```
(* This code is provided by the author, I. Green, to complement the subject paper. You may use the code freely. *)
(* Exercise any or all of the cases listed in Table 1 in
 the said paper. This code will reproduce all of the results in there. *)
(* Missing from this code is the implementation of the spectral moments as found
 in Ref.[10] because they produce results that are grossly in error. *)
(* The errors are caused by the approximated power spectrum,
sometimes dubbed as "the continuous power spectrum density," as derive in Ref. [11].
  That spectrum had been used in Ref. [10] and, since then, in many other papers. The errors sometimes approach 100%!
  These errors put the "approximated/continuous power spectrum," and the so-called "power law," in question.  *)
```

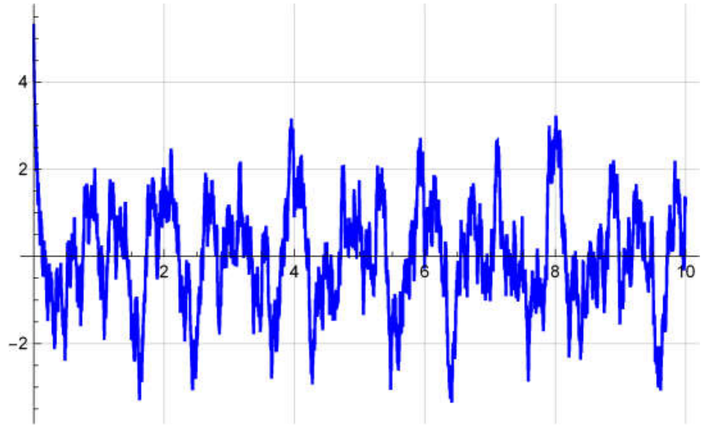
-----: DD = 1.5

-----: g = 1.5

-----: G = 1.

-----: q = 1

-----: n2 = 17



The EXACT Spectral Moments according to the IG paper (it is implied that LL−>Infinity);

These are entirely analytical, so there is NO CPU time consumed, at all!

-----: sm0 = m[0] = 1.49899

-----: sm2 = m[2] = 58 305.4

-----: sm4 = m[4] = 1.05914×10^{12}

-----: avg = 0.000191778

Spectral Moments by Differentiation
(the results are exact for this signal length, LL);
This method does consume CPU time! The larger n2, the larger the CPU time!

-----: m0 = 1.4997

-----: m2 = 58 305.9

-----: m4 = 1.05914×10^{12}

-----: cpu[seconds] = t1 - t0 = 27

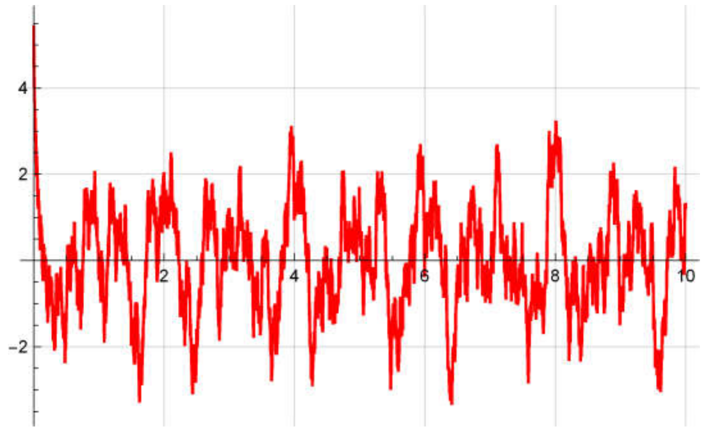
Relative difference between differentiation method and exact solution (the larger LL, the smaller the difference):

reldiff0=Abs[sm0-m0]/sm0 = 0.000475682 .....

reldiff2=Abs[sm2-m2]/sm2 = 9.27914×10^{-6} .....

reldiff4=Abs[sm4-m4]/sm4 = 1.47918×10^{-6} .....

-----: n2 = 26



The EXACT Spectral Moments according
to the IG paper (it is implied that LL–>Infinity);
These are entirely analytical, so there is
NO CPU time consumed, at all!

-----: sm0 = m[0] = 1.49997

-----: sm2 = m[2] = 2.24293×10^{6}

-----: sm4 = m[4] = 6.01752×10^{16}

-----: avg = 0.00019178

Spectral Moments by Differentiation
(the results are exact for this signal length, LL);
This method does consume CPU time! The larger n2, the larger the CPU time!

-----: m0 = 1.50069

-----: m2 = 2.24293×10^{6}

-----: m4 = 6.01752×10^{16}

-----: cpu[seconds] = t1 - t0 = 56

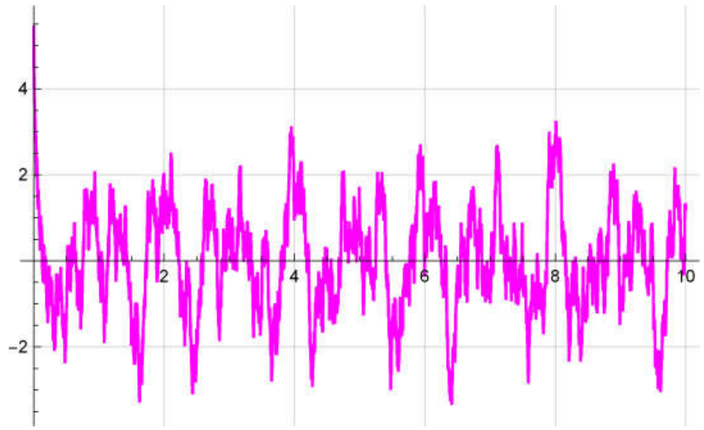
Relative difference between differentiation method and exact solution (the larger LL, the smaller the difference):

reldiff0=Abs[sm0-m0]/sm0 = 0.000475374 .....

reldiff2=Abs[sm2-m2]/sm2 = 2.14607×10^{-7} .....

reldiff4=Abs[sm4-m4]/sm4 = 5.12095×10^{-8} .....

-----: n2 = 34



The EXACT Spectral Moments according
to the IG paper (it is implied that LL–>Infinity);
These are entirely analytical, so there is
NO CPU time consumed, at all!

-----: sm0 = m[0] = 1.5

-----: sm2 = m[2] = 5.74849×10^{7}

-----: sm4 = m[4] = 1.013×10^{21}

-----: avg = 0.00019178

Spectral Moments by Differentiation
(the results are exact for this signal length, LL);
This method does consume CPU time! The larger n2, the larger the CPU time!

-----: m0 = 1.50071

-----: m2 = 5.74849×10^{7}

-----: m4 = 1.013×10^{21}

-----: cpu[seconds] = t1 - t0 = 89

Relative difference between differentiation method and exact solution (the larger LL, the smaller the difference):

reldiff0=Abs[sm0-m0]/sm0 = 0.000475365 .....

reldiff2=Abs[sm2-m2]/sm2 = 8.47025×10^{-9} .....

reldiff4=Abs[sm4-m4]/sm4 = 5.39462×10^{-9} .....

Fractal plots of all cases above