Gyroscopic and Support Effects on the Steady-State Response of a Noncontacting Flexibly Mounted Rotor Mechanical Face Seal

I. Green
The George W. Woodruff School of Mechanical Engineering,
Georgia Institute of Technology,
Atlanta, GA 30332-0405

The dynamic behavior of a noncontacting rotary mechanical face seal is analyzed. A closed-form solution is presented for the response of a flexibly mounted rotor to forcing misalignments which normally exist due to manufacturing and assembly tolerances. The relative misalignment between the rotor and the stator, which is the most important seal parameter, has been found to be time dependent with a cyclically varying magnitude. The relative response is minimum when support stiffness and damping are minimum. The gyroscopic couple is shown to have a direct effect on the dynamic response. This effect is enhanced at high speeds, and depending on the ratio between the transverse and polar moments of inertia, it can either decrease or increase the dynamic response. Its effect is most beneficial to seal performance when the rotor is a "short disk." A numerical example demonstrates that a flexibly-mounted rotor seal outperforms a flexibly mounted stator seal with regard to the total relative misalignment, the critical stator misalignment, and the critical speed.

Introduction

With the ever increasing demand for higher speeds, pressures, and temperatures in high performance rotating machinery, noncontacting sealing is essential for long life and reliable operation. Advances in analysis, development, and design of face seals in the past two decades have been impressive (Allaire, 1984). Yet, the vast majority of research, especially dynamic investigations, has concentrated on mechanical face seals whose flexibly mounted element is stationary (Etsion, 1982 and 1985). In a very recent study (Green, 1988) it was found that a mechanical face seal whose flexibly mounted element is rotating will be either inherently stable or conditionally unstable as a result of the gyroscopic couple. The nature of this couple was shown to depend upon the ratio between the transverse and polar moments of inertia of the rotor. If the seal is designed properly, i.e., if the rotor is a "short disk" so that the inertia ratio is kept below a critical value, dynamic stability is guaranteed at any speed (as opposed to the case of the flexibly mounted stator type seal (Green and Etsion, 1985)). On the other hand, if the rotor is a "long disk" so that the inertia is above the critical value, the seal may be either stable or unstable. However, most practical mechanical face seals fall into the category of "short disks." Therefore, it is quite unlikely that dynamic instability will ever occur in a real seal. Still, failure due to face contact or high leakage may result from a high relative misalignment between the two seal elements (stator and rotor) at steady state. This relative misalignment originates from stator and rotor misalignments normally resulting from manufacturing and assembly tolerances. This misalignment is directly affected by system parameters and operating conditions.

The problems associated with rotatry (i.e., flexibly mounted rotor) seals have been addressed by Nau (1981). A qualitative solution limited to rotor nutation angles much smaller than the stator misalignment has been presented by Metcalfe (1981). The effect of initial rotor misalignment upon the dynamic response has never been addressed.

This paper is a logical extension and completion of a

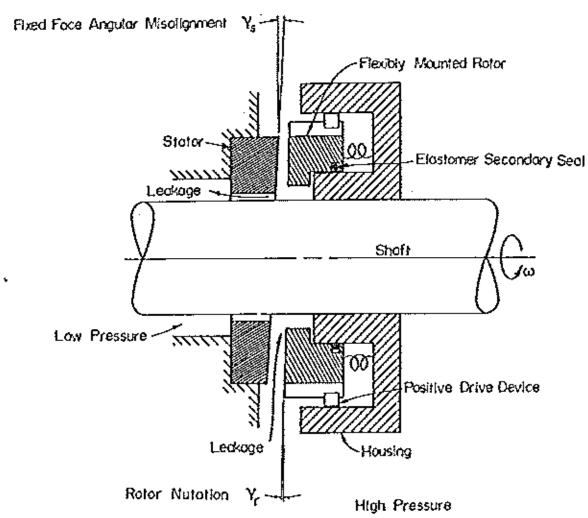

Fig. 1 Schematic of a flexibly mounted rotor face seal

Contributed by the Tribology Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS and presentated at the ASME/ASLE Joint Tribology Conference, Baltimore, Md., October 16-19, 1988. Manuscript received by the Tribology Division March 7, 1988. Paper No. 88-Trib-45.

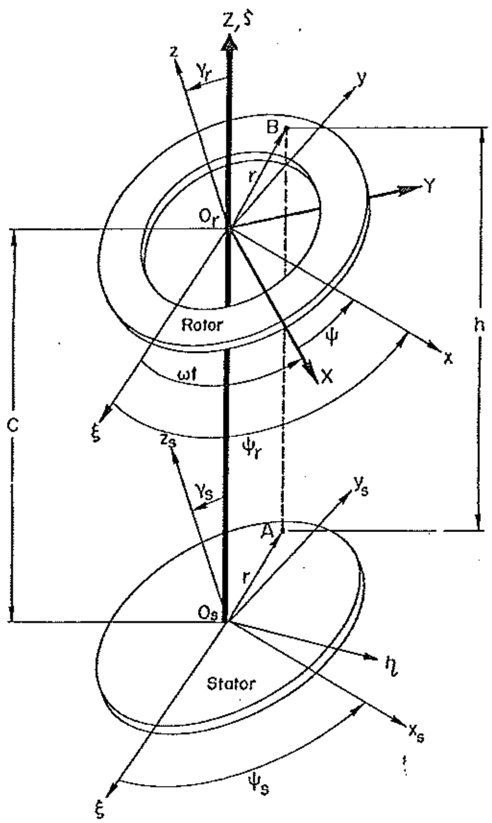

Fig. 2 Seal model and coordinate system

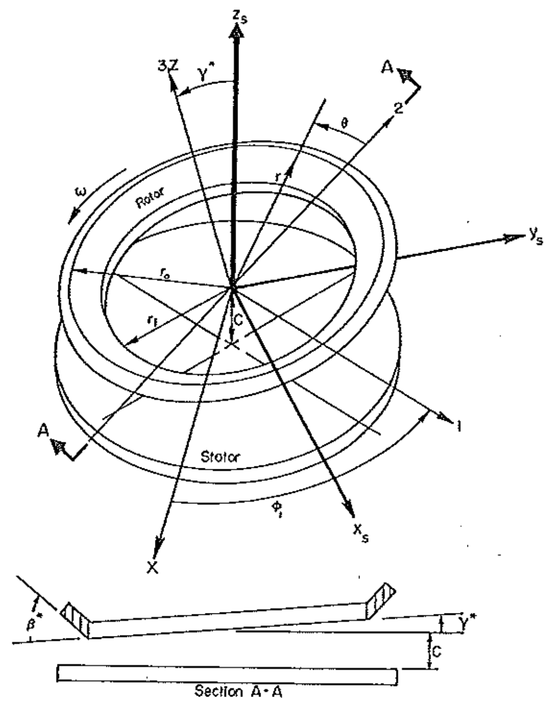

Fig. 3 Relative position between rotor and stator

previous work (Green, 1988) which focused on the stability problem. Here, the general steady-state problem where both rotor and stator are misaligned with respect to the axis of shaft rotation will be considered. The rotor response to these misalignments will be presented in a closed form analytical solution. The same effects that were found to greatly influence dynamic stability, namely, the gyroscopic couple and the flexible support characteristics, will draw our special attention.

Coordinate Systems and Equations of Motion

A noncontacting flexibly mounted rotor mechanical face seal is shown in Fig. 1. The rotor is supported by a circumferential spring and by a secondary seal, and it is driven by no more than two positive drive devices. (If the situation allows and the flexible support is capable of supporting the frictional torque at the interface (i.e., at the sealing dam), the drives should be eliminated altogether since they introduce an undesirable nonaxisymmetric character to the flexible support.) Five different coordinate systems are needed for the solution of the stated problem. The systems are shown in Figs. 2 and 3 and are described below:

1. Intertial reference system-$\xi\eta\zeta$
   - Fixed in space
   - ζ is the axis about which the shaft rotates
2. Rotating reference system-XYZ
   - Attached to the shaft and rotates with it at a speed ω
   - The directions of Z and ζ coincide
   - The angle between X and ξ is ωt
3. Rotor reference system-xyz
   - x is always in the XY plane
   - γ_r is the rotor nutation angle about x
   - y always points to the point in the rotor plane of maximum distance from the XY plane
   - The rotor is free to spin within xyz about z
   - The angle between X and x is the relative precession of the rotor, ψ
   - The angle between ξ and x is the absolute precession of the rotor, ψ_r. Hence,

$$\psi_r = \omega t + \psi \tag{1}$$

Nomenclature

- C_o = design clearance
- D^* = angular damping coefficient
- D = dimensionless angular damping coefficient, $D^*\omega C_o/Sr_o^4$
- I^* = rotor moments of inertia; transverse I^*, polar I_z^*
- I = dimensionless moment of inertia, $I^*\omega^2 C_o/Sr_o^4$
- K^* = angular stiffness coefficient
- K = dimensionless angular stiffness coefficient, K^*C_o/Sr_o^4
- m^* = rotor mass
- m = dimensionless mass, $m^*\omega^2 C_o/Sr_o^2$
- P = dimensionless pressure, p/S
- p = pressure
- R = dimensionless radial coordinate, r/r_o
- r = radial coordinate
- S = seal parameter, $6\mu\omega(r_o/C_o)^2(1-R_i)^2$
- β^* = face coning
- β = dimensionless coning, $\beta^* r_o/C_o$
- γ^* = misalignment
- γ = dimensionless misalignment, $\gamma^* r_o/C_o$
- μ = viscosity
- ω = shaft angular velocity

Subscripts

- cr = critical
- f = fluid film
- i = inner radius, or initial rotor misalignment
- I = rotor response to its own initial misalignment
- o = outer radius, or relative misalignment in the absence of γ_{ri}
- r = rotor
- s = stator, or flexible support

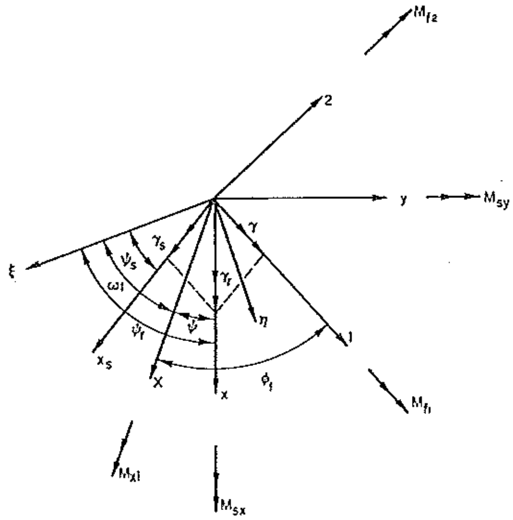

Fig. 4 Vector diagram of moments and tilts

4. Stator reference system-$x_s y_s z_s$
   - Fixed in space
   - x_s is parallel to the XY plane
   - γ_s is the stator misalignment about x_s
   - y_s always points to the point in the stator plane of minimum distance from the XY plane
   - ψ_s is the angular location of x_s measured from ξ
5. Relative reference system-123
   - Rotating reference as viewed by an observer placed on axis 1
   - 1 is always parallel to the $x_s y_s$ plane
   - γ is the relative misalignment measured between rotor and stator normals, z and z_s, respectively
   - 2 always points to the point in the rotor plane of maximum distance between rotor and stator
   - The angle between X and 1 is the relative precession of axis 1, ϕ_1
   - For small nutation and misalignment angles, the following vector relationship applies:

$$\vec{\gamma} = \vec{\gamma}_r - \vec{\gamma}_s \tag{2}$$

Equation (2) is valid for any practical mechanical face seal since the axial clearance, C (see Figs. 2 and 3), is very small compared to any radial distance to the sealing dam. Therefore, tilts are very small, and all reference systems can be described in the planar vector diagram of Fig. 4. The vector relationship of equation (2) is shown in the parallelogram along with the various applied moments that act on the flexibly-mounted rotor. One such moment, M_{XI}, is due to the rotor's own initial misalignment, γ_{ri} (prior to attachment), as can be expected from an imperfect spring or secondary seal support. The motion of the rotor within the housing produces an applied moment, M_s, which originates from the flexible support (expressed in the rotating reference xyz), while the relative motion between the rotor and stator produces the fluid film moment, M_f (expressed in the relative system 123). These moments along with the equations of motion have been derived in detail by Green (1988). That work demonstrates that as long as the analysis is limited to small perturbations, the equation of motion in the axial mode is linear, homogeneous, and decoupled from the equations in the angular mode. The treatment of this equation in that work is complete and will not be repeated here. However, the nonhomogeneous equations of motion in the angular mode remain to be solved. These equations read:

$$I(\ddot{\gamma}_r - \dot{\psi}_r^2\gamma_r) + I_z\dot{\psi}_r\gamma_r = -K\gamma_r - D\dot{\gamma}_r + K_s\gamma_{ri}\cos\psi + \gamma_s\left(K_f\cos\rho' - \frac{1}{2}D_f\sin\rho'\right) \tag{3a}$$

$$I(\ddot{\psi}_r\gamma_r + 2\dot{\psi}_r\dot{\gamma}_r) - I_z\dot{\gamma}_r = -D\dot{\psi}\gamma_r - \frac{1}{2}D_f\gamma_r - K_s\gamma_{ri}\sin\psi - \gamma_s\left(K_f\sin\rho' + \frac{1}{2}D_f\cos\rho'\right) \tag{3b}$$

The last term on the left-hand side of both equations, which involves I_z, is a gyroscopic term. These terms are inherent in the problem because the dynamic moment is written for a misaligned rotating element, that is, the flexibly mounted rotor (Green and Etsion (1986a)). The overall stiffness and damping coefficients, K and D, respectively, are given by

$$K = K_s + K_f \tag{4a}$$

$$D = D_s + D_f \tag{4b}$$

where the subscripts s and f designate the support and the fluid film, respectively. The above equations are nondimensional (see Nomenclature for normalization). The various normalized angular coefficients are summarized in the Appendix. The angles ρ' and ρ (used later) are simply

$$\rho' = \psi_r - \psi_s \tag{5a}$$

$$\rho = \phi_1 - \psi \tag{5b}$$

Steady-State Response

The equations of motion (3) are expressed in the Eulerian system, xyz, and are obviously nonlinear. However, when these equations are transformed to the inertial system, $\xi\eta\zeta$, with the simplification that misalignment and nutation angles involved are very small, they become linear (though coupled) equations (Green, 1988). The linear nature of the equations enables us to identify the forcing functions and solve for each forcing function separately. The responses can then be combined, employing the superposition principle, to produce the complete steady-state response. The two forcing functions involved originate with the fixed stator misalignment, γ_s, and the initial rotor misalignment γ_{ri}.

Rotor Response to Stator Misalignment. In this section we assume that the rotor is initially perfectly aligned, i.e., $\gamma_{ri} = 0$. Since the stator misalignment, γ_s, is fixed in space, the rotor response, γ_{rs}, must also be fixed in space. Explicitly, the time derivatives of the nutation and the absolute precession angles equal zero:

$$\ddot{\gamma}_{rs} = \dot{\gamma}_{rs} = 0$$

$$\ddot{\psi}_r = \dot{\psi}_r = 0$$

where ωt is normalized time. If the rotor maintains an absolute precession rate of zero ($\dot{\psi}_r = 0$) equation (1) results in $\dot{\psi} = -1$. Due to the kinematical constraint imposed on the rotor in the form of a universal joint (Green and Etsion, 1986a), the spin rate is the negative of the relative precession rate. This means that while the housing rotates about axis Z at a speed ω, the rotor spins about axis z of Fig. 2 at the same speed. At the same time the rotor precesses (whirls) in a direction opposite to that of the housing (i.e., the negative Z direction). Since the magnitudes of all angular velocities are equal the motion can be called "synchronous retrograde precession" or "synchronous backward whirl." The vector diagram of Fig. 5 describes the kinematic relationship between the parameters involved in this case. Substituting the identities above into equation (3) and rearranging, we have

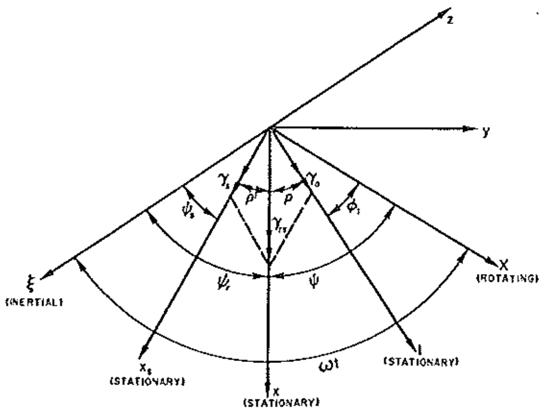

Fig. 5 Vector diagrams of the rotor response to the stator misalignment

$$k\gamma_{rs} = \left(K_f\cos\rho' - \frac{1}{2}D_f\sin\rho'\right)\gamma_s \tag{6a}$$

$$\left(D_s + \frac{1}{2}D_f\right)\gamma_{rs} = \left(K_f\sin\rho' + \frac{1}{2}D_f\cos\rho'\right)\gamma_s \tag{6b}$$

Adding the squares of right and left hand sides defines the absolute transmissibility of the system

$$\left(\frac{\gamma_{rs}}{\gamma_s}\right)^2 = \frac{K_f^2 + \frac{1}{4}D_f^2}{K^2 + \left(D_s + \frac{1}{2}D_f\right)^2} \tag{7}$$

Note that inertia is not a part of the solution due to the above kinematical conditions which cause all of the dynamic moments to vanish, i.e., this is a static response of the rotor to a static forcing function. Therefore, axis x in Fig. 5 is stationary. Of great importance is the relative misalignment, γ, between the rotor and stator. This misalignment directly influences the leakage from the seal and determines whether touchdown of the faces occurs. Therefore, this misalignment should be minimized. Since γ_s and γ_{rs} are stationary, equation (2) implies that γ (or in this case γ_0) must also be stationary as indicated on axis 1 in Fig. 5. Using equation (2) and Fig. 5, we have

$$\gamma\cos\rho = \gamma_r - \gamma_s\cos\rho' \tag{8a}$$

$$\gamma\sin\rho = \gamma_s\sin\rho' \tag{8b}$$

Substituting equations (8) in equations (6) results in

$$K_s\gamma_{rs} = -\left(K_f\cos\rho + \frac{1}{2}D_f\sin\rho\right)\gamma_0 \tag{9a}$$

$$D_s\gamma_{rs} = \left(K_f\sin\rho - \frac{1}{2}D_f\cos\rho\right)\gamma_0 \tag{9b}$$

where in this case we have substituted γ_o and γ_{rs} for γ and γ_r, respectively. Repeating the same process that follows equations (6), we get

$$\left(\frac{\gamma_0}{\gamma_{rs}}\right)^2 = \frac{K_s^2 + D_s^2}{K_f^2 + \frac{1}{4}D_f^2} \tag{10}$$

Multiplying equation (7) by equation (10) gives the relative transmissibility

$$\left(\frac{\gamma_0}{\gamma_s}\right)^2 = \frac{K_s^2 + D_s^2}{K^2 + \left(D_s + \frac{1}{2}D_f\right)^2} \tag{11}$$

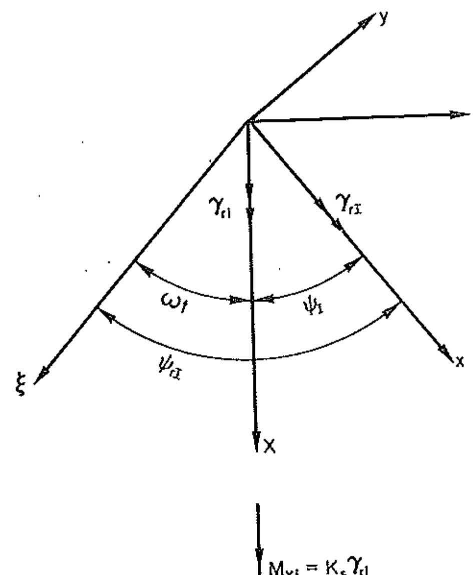

Fig. 6 Vector diagram of the rotor response to its own initial misalignment

It is quite obvious that in order to minimize γ_o we require that K_s and D_s be small compared to K_f and D_f, indicating that a very soft support is preferable. A hypothetical elimination of K_s and D_s results in $\gamma_{rs} = \gamma_s$ and $\gamma_0 = 0$ as can be easily verified from equations (7) and (11) (note that $K = K_s + K_f$, as given by equation (4a)).

Rotor Response to Its Own Initial Misalignment. In this section we assume that the stator is perfectly aligned, i.e., $\gamma_s = 0$ where γ_{ri} is the initial rotor misalignment. Since the moment $M_{Xi} = K_s\gamma_{ri}$ is constant in the rotating reference XYZ, the rotor response, γ_{rI}, to this forcing function must be of the same nature. This is, γ_{rI} is constant in magnitude and rotates with the same frequency as M_{Xi}. This situation is illustrated in Fig. 6, where the following mathematical identities apply:

$$\gamma_{rI} = \text{const}; \quad \dot{\psi}_{rI} = 1; \quad \dot{\psi}_I = 0$$

Substituting these identities in the equations of motion (3) and rearranging results in

$$(I_z - I + K)\gamma_{rI} = K_s\gamma_{ri}\cos\psi_I \tag{12a}$$

$$\frac{1}{2}D_f\gamma_{rI} = -K_s\gamma_{ri}\sin\psi_I \tag{12b}$$

Again, adding the squares of the right and left-hand sides of equations (12) and simplifying, results in the response

$$\gamma_{rI} = \frac{K_s\gamma_{ri}}{\left[(I_z - I + K)^2 + \frac{1}{4}D_f^2\right]^{1/2}} \tag{13a}$$

Solving for the relative precession angle, ψ_I, we have

$$\tan\psi_I = -\frac{\frac{1}{2}D_f}{I_z - I + K} \tag{13b}$$

where the negative sign indicates that the response lags the forcing function. Once again we see that as K_s decreases ("soft" flexible support) the response, γ_{rI}, decreases. Eliminating γ_{rI} altogether is, of course, even more desirable.

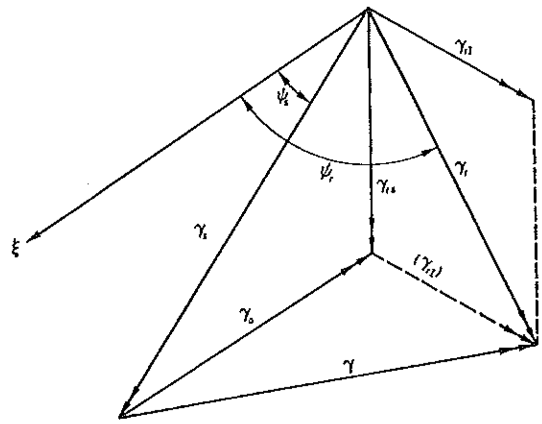

Fig. 7 Vector representation of the complete response

The response γ_{rl} as expressed in equation (13*a*), is also the relative misalignment between rotor and stator since in this case $\gamma_s = 0$.

The Complete Response. The complete steady-state response of the rotor can now be obtained by utilizing the superposition principle, i.e., adding vectorially the two separate responses γ_{rs} and γ_{rl}

$$\vec{\gamma}_r = \vec{\gamma}_{rs} + \vec{\gamma}_{rl} \tag{14}$$

Since $\vec{\gamma}_{rs}$ is fixed in space, while $\vec{\gamma}_{rl}$ rotates at the speed ω, the over all response, $\vec{\gamma}_r$, is a rotating vector with a time varying frequency, $\dot{\psi}_r$. The magnitudes of both $\dot{\psi}_r$ and $\vec{\gamma}_r$ vary cyclically with a constant frequency ω.

A more important parameter is the relative misalignment, γ, between rotor and stator. By adding vectorially the relative responses γ_o and γ_{rl} obtained from the solution of the two cases $\gamma_{ri} = 0$ and $\gamma_s = 0$, respectively, we have

$$\vec{\gamma} = \vec{\gamma}_0 + \vec{\gamma}_{rl} \tag{15}$$

where by equation (2) and for $\gamma_{ri} = 0$ we have

$$\vec{\gamma}_0 = \vec{\gamma}_{rs} - \vec{\gamma}_s \tag{16}$$

Again, since the vector $\vec{\gamma}_o$ is fixed in space and the vector $\vec{\gamma}_{rl}$ rotates at the speed ω, the overall relative response, $\vec{\gamma}$, is a rotating vector of varying relative frequency, $\dot{\phi}$. The magnitudes of both $\dot{\phi}$ and $\vec{\gamma}$ vary cyclically at a constant frequency ω. All the vector operations as expressed in equation (2) and equations (14) through (16) are shown in Fig. 7. With the aid of this figure, equation (15) can be expressed in a scalar form

$$\gamma^2 = \gamma_o^2 + \gamma_{rl}^2 + 2\gamma_0\gamma_{rl}\cos\omega\tau \tag{17}$$

where τ is some reference time measured from the instant at which the rotating vector $\vec{\gamma}_{rl}$ passes over the stationary $\vec{\gamma}_o$. Equation (17) once again demonstrates the time dependence of the relative misalignment γ. Obviously, the maximum value occurs when $\tau = 0$, in which case

$$(\gamma)_{\max} = \gamma_o + \gamma_{rl} \tag{18}$$

The minimum value, however, occurs when $\omega\tau = \pi$. Hence,

$$(\gamma)_{\min} = |\gamma_0 - \gamma_{rl}| \tag{19}$$

The magnitudes of the responses γ_{rs}, γ_{rl}, and γ_o and their positions relative to each other determine whether the time dependent absolute and relative responses, γ_r and γ, respectively, are leading or lagging the stationary forcing function γ_s. While a lagging response may exist over an entire shaft cycle, a leading response can exist only over a portion of a shaft cycle. From a practical standpoint this is of lesser importance.

Discussion of Results

The relative misalignment, γ, as given by equation (17) is of prime importance in the determination of seal performance. For example, leakage is proportional to γ^2 (Etsion and Sharoni, 1980), and face touchdown occurs when the maximum relative misalignment given by equation (18) is too high. The only way to eliminate γ altogether is to eliminate stator and rotor misalignments, γ_s and γ_{ri}, respectively. This, however, is practically impossible. The other alternative is to require the responses, γ_o and γ_{rl}, to these misalignments to vanish. As can be seen from equations (11) and (13*a*) this goal is achieved if K_s and D_s equal zero. However, such values in mechanical seals are once again impractical since some stiffness and possibly some damping do exist in a flexible support which consists of a circumferential spring and a secondary seal. Since in a practical seal some misalignments as well as some stiffness and damping of the support must exist, we will examine in this section the other parameters which govern the relative misalignment.

First, by observing equations (11) and (13*a*) we see that maximum fluid stiffness and damping, K_f and D_f, respectively, reduce the two responses, γ_o and γ_{rl}. The conditions under which K_f and D_f are maximum have been discussed elsewhere (Green, 1987). Generally, there exists an optimal face coning which maximizes fluid film stiffness, while flat faces (no coning) maximize fluid film damping. The best coning angle will therefore depend on system parameters such as pressure drop, viscosity, geometry, etc. For example, in a high pressure seal it is reasonable to maximize the fluid stiffness by having optimal coning since the stiffness is directly proportional to the pressure drop. In a low pressure seal for a high fluid viscosity, a small amount of coning to maximize fluid damping would probably be the preferable configuration. (Maximum values of K_f and D_f have also been found to be preferable with regard to dynamic stability (Green, 1988)).

The gyroscopic effect is very interesting. As we have seen (equations (3)), the inherent gyroscopic term comes into effect through the polar moment of inertia, I_z. Using a relationship between the transverse and the polar moments of inertia, I and I_z, respectively,

$$c = I/I_z$$

and using dimensional parameters (see nomenclature), equation (13*a*) takes the form

$$\gamma_{rl}^* = \gamma_{ri}^* \frac{K_s^*}{\left\{\left[\left(\frac{1}{c} - 1\right)I^*\omega^2 + K^*\right]^2 + \left(\frac{1}{2}D_f^*\omega\right)^2\right\}^{1/2}}$$

Now the gyroscopic effect is imbedded in the parameter c. For cylindrical rigid bodies, c is bounded from below by 1/2. To minimize the response γ_{rl}^* we require c to be minimum, that is, $c = 1/2$. This value corresponds to a very short disk, i.e., the rotor should have a small length to radius ratio. Consequently, higher speeds are preferable because they further reduce the response. As c increases, the response γ_{rl}^* increases. Still, as long as c is in the range $1/2 \le c < 1$ the speed, ω, has a positive effect in reducing γ_{rl}^*. When $c = 1$, note that the response γ_{rl}^* is independent of inertia as is the case for the response γ_0 (see Equation (11)). However, if $c > 1$ the speed changes its role and contributes to an increase in γ_{rl}^*. This range should therefore be avoided.

Of interest is to compare the performance of the flexibly mounted rotor seal with that of the flexibly-mounted stator seal (Green and Etsion, 1985). The two seals will be referred to as FMR and FMS, respectively. First, the critical stator misalignment that causes face contact needs to be determined.

Table 1 Performance comparison between the flexibly mounted rotor (FMR) and the flexibly mounted stator (FMS)

	FMR	FMS
Static transmissibility	$\frac{\gamma_0}{\gamma_s} = 0.071$	$\frac{\gamma_{sl}}{\gamma_{sl}} = 0.061$
Dynamic transmissibility	$\frac{\gamma_{rl}}{\gamma_{ri}} = \begin{cases} 0.055@c=1/2 \\ 0.066@c=3 \end{cases}$	$\frac{\gamma_0}{\gamma_r} = 0.080$
Maximum relative misalignment	$(\gamma)_{\max} = \begin{cases} 0.126@c=1/2 \\ 0.137@c=3 \end{cases}$	$(\gamma)_{\max} = 0.141$
Critical misalignment	$(\gamma_s)_{cr} = \begin{cases} 16.8@c=1/2 \\ 16.7@c=3 \end{cases}$	$(\gamma_r)_{cr} = 14.8$
Critical speed	$\omega_{cr} = \begin{cases} \infty@c=1/2 \\ 8.4\omega=8.4\times10^3\text{rad/s} \ @c=3 \end{cases}$	$\omega_{cr} = 5.9\omega = 5.9\times10^3\text{rad/s}$

As explained in the previous work, if the coning angle is greater than a critical value, face contact is likely to occur on the inner radius of the seal, i.e., $1-(\gamma)_{\max}\ R_i=0$, where $R_i=r_i/r_o$ (see Fig. 3). By equation (18) we have

$$\gamma_0+\gamma_{rl}=\frac{1}{Ri} \tag{19}$$

The relative misalignment is related to the stator misalignment by the relative transmissibility, $T=\gamma_0/\gamma_s$, given by the square root of equation (11). Substituting $T(\gamma_s)_{cr}$ for γ_0 in equation (19) results in the critical stator misalignment.

$$(\gamma_s)_{cr}=\frac{\frac{1}{Ri}-\gamma_{rl}}{T} \tag{20}$$

It can be seen that the parameters which minimize γ_{rl} and T also maximize $(\gamma_s)_{cr}$. A maximum value is preferable from a design and manufacturing standpoint. As with the case of the FMS, the analysis here is limited to small perturbations about the equilibrium position, i.e., design clearance and parallel faces. When face contact occurs the motions involved are not small. However, as has been found for the FMS case (Green and Etsion, 1986b), the prediction based on small perturbations results in a conservative value of the critical misalignment. Hence, assuming characteristics similar to those of the FMS case, the result given in equation (20) can be used with a certain degree of safety.

We now examine the same "typical" seal, design parameters, and operating conditions as in the case of the FMS seal:

Seal outer radius, r_0	0.04 m
Radius ratio, R_i	0.8
Design clearance, C_o	10^{-5}m
Face taper (cone height)	2.5×10^{-5}m
Rotor mass, m*	1 kg
Shaft speed, ω	10^3 rad/s
Pressure differential, p_o-p_i	5×10^5Pa
Fluid viscosity, μ(water at 60°C)	0.5 mPas
Support axial stiffness, K_s^*	5×10^5N/m
Support axial damping, D_s^*	300 Ns/m

The corresponding dimensionless parameters for this example are:

Coning angle, β	12.5
Mass, m	3.25×10^{-3}
Inertia, I	1.63×10^{-3}
Pressure differential, P_o-P_i	0.26
Support axial stiffness, K_{s33}	1.63×10^{-3}
Support angular stiffness, K_s	8.14×10^{-4}
Support axial damping, D_{s33}	9.76×10^{-4}
Support angular damping, D_s	4.88×10^{-4}
Fluid film angular stiffness, K_f	0.118×10^{-1}
Fluid film angular damping, D_f	0.83×10^{-2}

The comparison is summarized in Table 1. The first comparison is done on the static response. The FMR has a slightly higher static transmissiblity than the FMS seal, which is due to the support damping. If the support damping is eliminated altogether (which is recommended anyway) the two configurations result in exactly the same transmissibility. The next comparison involves inertia effects and the dynamic response. The dynamic transmissibility of the FMR is considerably smaller than that for the FMS at $c=1/2$ ("short rotor") and is also smaller even at $c=3$ (which corresponds to a "long rotor," whose length is about 5.5 times its radius which would be a "bad" design). It is apparent that for all parameters besides the static transmissibility the FMR seal outperforms the FMS seal. If the forcing functions for the two cases are equal, then the FMR seal has a lower maximum relative misalignment, resulting in a higher critical stator misalignment. The results, 0.126 or 0.137, for the maximum relative misalignment assure that the small perturbation assumption was justified even more in this case. The critical speed data have been calculated using the analysis by Green (1988). A "short rotor" guarantees dynamic stability for the FMR case, while even a long rotor still provides a critical speed substantially higher than that of the FMS case.

Concluding Remarks

The steady state response of a noncontacting flexibly-mounted rotor mechanical face seal has been investigated analytically. This response is an outcome of the stator misalignment and the initial rotor misalignment. The response has been presented in the form of relative and absolute transmissibilities which enable the determination of a maximum relative misalignment and of a critical stator misalignment. These two parameters are very important to seal performance, i.e., leakage and whether face contact occurs. Rotor nutation and relative misalignment have been found to be time dependent. For improved steady-state response and dynamic stability, the following are recommended:

- The flexible support should be undamped and "soft."
- The flexibly-mounted rotor should be a "short-disk."

A numerical example has been presented comparing FMR and FMS seals. In this example, it has been shown that the FMR seal performance exceeds that of the FMS seal, and it is suggested that such a comparison be made regularly during the design process.

Based on previous experience, it seems intuitively that a seal whose stator and rotor are both flexibly-mounted would result in even better performance. The reason for this is that if the rotor is designed properly ("short disk") the gyroscopic effect will tend to align it with respect to the shaft axis of rotation, and this tendency increases with speed. The stator, however, will try to track an aligned rotor, resulting in minimum relative misalignment. An even more daring configuration is suggested. If the two seal faces are flexibly-mounted and rotating, and if both are "short disks," the gyroscopic effect will align the two elements. Such a configuration is not inconceivable in high performance rotating machinery.

Acknowledgment

This paper is based upon the work supported by the National Science Foundation under Grant Number MSM-8619190. This support is gratefully acknowledged.

References

Allair, P. E., 1984, "Noncontacting Face Seals for Nuclear Applications—A Literature Review," *Lubrication Engineering*, Vol. 40, No. 6, pp. 344–351.

Etsion, I., and Sharoni, A., 1980, "Performance of End-Face Seals with Diametral Tilt and Coning-Hydrostatic Effects," *ASLE Transactions*, Vol. 23, No. 3, pp. 279–288.

Etsion, I., 1982, "A Review of Mechanical Face Seal Dynamics," *The Shock and Vibration Digest*, Vol. 14, No. 3, pp. 9–14.

Etsion, I., 1985, "Mechanical Face Seal Dynamics Update," *The Shock and Vibration Digest*, Vol. 17, No. 4, pp. 11–15.

Green, I., and Etsion, I., 1985, "Stability Threshold and Steady-State Response of Noncontacting Coned-Face Seals," *ASLE Transactions*, Vol. 28, No. 4, pp. 449–460.

Green, I., and Etsion, I., 1986a, "A Kinematic Model for Mechanical Seals with Antirotation Locks or Positive Drive Devices," ASME Journal of Tribology, Vol. 108, No. 1, pp. 42–45.

Green, I., and Etsion, I., 1986b, "Nonlinear Dynamic Analysis of Noncontacting Coned-Face Mechanical Seals," *ASLE Transactions*, Vol. 29, No. 3, pp. 383–393.

Green, I., 1987, "The Rotor Dynamic Coefficients of Coned-Face Mechanical Seals with Inward or Outward Flow," *Transaction of ASME* Journal of Tribology, Vol. 109, No. 1, pp. 129–135.

Green, I., 1988, "Gyroscopic and Damping Effects on the Stability Threshold of a Noncontacting Flexibly-Mounted Rotor Mechanical Face Seal," the Second International Symposium on Transport Phenomena, Dynamics, and Design of Rotating Machinery, Honolulu, Hawaii, pp. 77–97.

Metcalfe, R., 1981, "Dynamic Tracking of Angular Misalignment in Liquid-Lubricated End-Face Seals," *ASLE Transactions*, Vol. 24, No. 4, pp. 509–516.

Nau, B. S., 1981, "Vibration and Rotary Mechanical Seals," *Tribology International*, pp. 55–59.

APPENDIX

Angular Stiffness and Damping Coefficients

Fluid Film. The nondimensional axial and angular rotor dynamic coefficients have been provided by Green (1987). In the present work only the angular coefficients are of interest:

$$k_f = \pi(P_o - P_i)(\beta R_i - 1)\left[\frac{1-R_i^2}{4+2\beta(1-R_i)}\right]^2$$

$$D_f = 2\pi R_m^3 G_o \ @ \ R_m = \frac{1+R_i}{2}$$

where

$$G_o = \frac{\ln[1+\beta(1-R_i)] - 2\dfrac{\beta(1-R_i)}{2+\beta(1-R_i)}}{\beta^3(1-R_i)^2}$$

and for flat faces ($\beta = 0$) G_o simplifies to

$$G_o = \frac{1-R_i}{12}$$

(The cross coupled angular coefficient has already been included in the equations of motion (3)). The dimensional coefficient can be obtained using the normalizing factors provided in the Nomenclature.

Flexible Support. The dimensional angular stiffness and damping coefficients, K_s^* and D_s^*, respectively, can be obtained from the corresponding axial coefficients, K_{s33}^* and D_{s33}^*, using the following transformation (Green and Etsion, 1985):

$$K_s^* = \frac{1}{2} K_{s33}^* r_s^2$$

$$D_s^* = \frac{1}{2} D_{s33}^* r_s^2$$

where r_s is the radial dimension of the flexible support element. If more than one element exists, then the above transformation should be used for each element separately. The nondimensional coefficients are defined in the Nomenclature.

DISCUSSION

I. Etsion[1]

This paper is a valuable contribution to the seal literature. It may be of great help to both designers and users in selecting a seal concept, i.e., FMR or FMS, for some given application and operating conditions. Comparisons such as in Table 1 are very useful for this purpose. This comparison could be generalized by evaluating the corresponding transmissibility equations of the FMR and the FMS, i.e., equations (11) and (13*a*) in the present paper and equations (31*a*) and (37), respectively, in Green and Etsion (1985). Assuming $D_s \ll D_f$ (which is normally the case) and searching for the condition that makes the transmissibility of the FMS less than that of the FMR one obtains

$$D_s \neq 0 \tag{1}$$

for the static transmissibility, and

$$D_s^2 < \frac{(K-I)^2+(D_f/2)^2}{(K+I)^2+(D_f/2)^2} K_s^2 - I^2 \tag{2}$$

for the dynamic transmissibility (at $c = 1/2$).

From (1) is is clear that the static transmissibility of the FMS is always smaller than that of the FMR. Condition (2) can be easily met at low speed, ω, when $I \ll K$ and $D_s < K_s$ (see Nomenclature for definition of dimensionless parameters). As the shaft speed, ω, increases the dimensionless moment of inertia, I, increases too. Eventually at a certain speed $\omega = \omega^*$ the right-hand side of (2) vanishes. If ω is further increased so that $\omega > \omega^*$ condition (2) no longer holds and the dynamic transmissibility of the FMS exceeds that of the FMR. Since the static transmissibility is speed independent one may conclude that below a certain speed ω^* the relative misalignment of the FMS is always smaller than that of the FMR and, hence, the FMS is preferable. Above a certain critical speed (higher than ω^*) the relative misalignment of the FMS becomes larger than that of the FMR and the FMR concept is preferable. There may, however, be other problems resulting from high speed rotating flexible support components that have to be resolved before the full benefit of the FMR concept at high speeds can be realized.

It is interesting to note that the relative misalignment of the FMR decreases at high speeds. Intuitively one may think that centrifugal forces will tend to align the FMR with the rotating shaft and, hence, prevent alignment with the tilted stator, causing increasing relative misalignment with speed. However, the special kinematic constraints of the FMR prevent any such centrifugal effects. The author should elaborate more on this point to enhance the clarity of this valuable paper.

Author's Closure

The author wishes to thank Dr. Etsion for his interest in the paper and for his constructive remarks.

The author completely concurs with Dr. Etsion's observation that there is a speed, ω^*, above which the benefits of the FMR seal are perceived. However, analytical determination of ω^* is not feasible. Therefore, a comparison between the FMR and FMS seals, based on the criteria in Table 1, can be performed numerically or graphically as shown in Fig. 8. This figure presents the static, dynamic, and total transmissibilities as a function of shaft speed, ω, for the typical seal where $c = 1/2$. (Table 1 provides numerical data for this seal at a speed of 1000 rad/s.) The total transmissibility equals the maximum relative misalignment under the conditions of the comparison

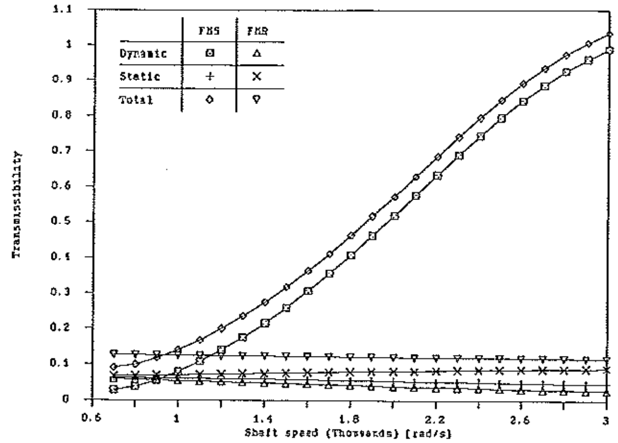

Fig. 8 Transmissibility comparison between the flexibly mounted rotor (FMR) and the flexibly mounted stator (FMS) seals

of Table 1. As noted in the paper, and also by Dr. Etsion, Fig. 8 confirms that the FMS seal always has a lower static transmissibility than the FMR seal. Nevertheless, the two static transmissibilities are of the same order of magnitude. However, as speed increases, the FMR seal substantially outperforms the FMS seal. The dynamic response of the FMR seal decreases as speed increases due to the gyroscopic effect (see section "Discussion of Results"), while the dynamic response of the FMS seal increases with speed under the same conditions. At 3000 rad/s there is an order of magnitude difference between the dynamic and the total transmissibilities of the two seals. The values of ω^* for the dynamic, and total transmissibilities are 903 rad/s, and 938 rad/s, respectively. It is worthwhile emphasizing that the FMR seal is unconditionally stable when $c = 1/2$, as opposed to the FMS seal.

The relative misalignment, γ_o, which is a direct outcome from the static transmissibility (see equation (11)) is rather increasing with speed as can be seen in Fig. 8. For the typical seal under consideration, values for K_s^*, D_s^*, K_f^*, and D_f^* have been determined to be 400 N•m, 0.24 N•m•s, 5791 N•m, and 4.082 N•m•s, respectively. Rewriting equation (11) using dimensional parameters, yields

$$T_s = \frac{\gamma_0{}^*}{\gamma_s{}^*} = \left[\frac{K_s^{*2} + (D_s^* \omega)^2}{(K_s^* + K_f^*)^2 + \left(D_s^* \omega + \dfrac{1}{2} D_f^* \omega \right)^2} \right]^{1/2}$$

The derivative, $\partial T_s / \partial \omega$, for the above parameters results in positive values for a speed range 0 to 10,000 rad/s. This indicates that T_s is monotonically increasing with ω. But this monotonic behavior is attributable to the relative magnitudes of the parameters under consideration rather than to the gyroscopic effect ("centrifugal force") which does not exist for this static forcing function. To explain this phenomenon we resort to a system which is kinematically equivalent to the problem in hand, as it similarly responds to a static forcing function.

Consider the system of Fig. 9, where a disk is mechanically engaged to a rotating shaft by means of a universal joint. A stationary pin, supported by a spring, is brought into contact with the rotating disk, and then further pushed to cause the disk to tilt an amount, γ, measured between the axis of shaft rotation, Z, and axis z which is normal to the disk. The system xyz can only tilt about axis x. The angular velocity of xyz is

$$\vec{\omega}_c = \dot{\gamma} \hat{x}$$

The angular velocity of the disk relative to xyz is the spin $\dot{\phi}$. However, due to the kinematical constraint of the universal

[1]Technion, Haifa 32000 Israel.

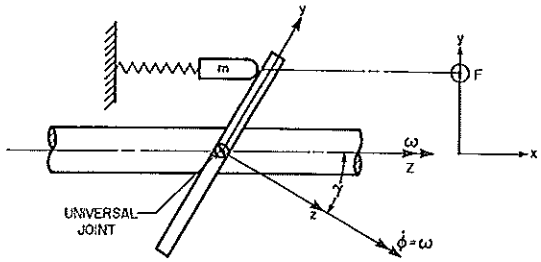

Fig. 9 Pin and disk system

joint (see Green and Etsion (1986a)) the transmissibility $T = 1$, or $\dot{\phi} = \omega$. The absolute angular velocity is then

$$\vec{\lambda} = \vec{\omega}_c + \dot{\phi}\hat{z} = \dot{\gamma}\hat{x} + \omega\hat{z}$$

The relative angular momentum is simply

$$\vec{L} = I\dot{\gamma}\hat{x} + I_z\omega\hat{z}$$

The dynamic moment for a nontranslating disk is given by

$$\vec{T} = \frac{\partial \vec{L}}{\partial t} + \omega_c \times \vec{L}$$

Hence,

$$\vec{T} = I\ddot{\gamma}\hat{x} - I_z\omega\dot{\gamma}\hat{y}$$

The equations of motion are obtained by equating the dynamic and applied moments. The only applied moment results from the contacting force, F, between the pin and the disk. Hence, the equations of motion are

$$I\ddot{\gamma} = FR$$

$$-I_z\omega\dot{\gamma} = 0$$

where R is the radial distance to the contact point. From the last equation we see that the gyroscopic moment vanishes where we have $\dot{\gamma} = 0$. Therefore, $\ddot{\gamma} = 0$, which results in $F = 0$. This result indicates that although the disk is spinning the spring remains uncompressed, regardless of the shaft speed, ω. To conclude, the reason for the vanishing dynamic moment originates with the kinematical constraint which enables the disk to spin about its own axis rather than the shaft axis.

If you are planning
To Move, Please
Notify The
ASME-Order Dep't
22 Law Drive
Box 2300
Fairfield, N.J. 07007-2300

Don't Wait!
Don't Miss An Issue!
Allow Ample Time To
Effect Change.

Change of Address Form for the Journal of Tribology

Present Address – Affix Label or Copy Information from Label

Print New Address Below

Name ____________________
Attention ____________________
Address ____________________
City __________ State or Country __________ Zip __________