Steady-State Analysis of Mechanical Seals With Two Flexibly Mounted Rotors

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The dynamic behavior of a mechanical face seal with two flexibly mounted rotors is investigated. The equations of motion are derived using linearized rotor dynamic coefficients to model the dynamic behavior of the fluid film. The equations are shown to be linear in the inertial reference with harmonic forcing functions which result from the initial misalignment of the flexible supports. A method for obtaining the steady-state response in the system is derived by transforming the equations of motion into reference frames which rotate with the shafts. The resulting equations contain constant forcing functions and can be readily solved for the magnitude of the steady-state response. The method presented allows a rapid determination of the steady-state misalignment of a seal without resorting to numerical modeling.

Introduction

Most of the literature dealing with the dynamic behavior of mechanical seals (Etsion, 1982, 1985, and 1989) has been focused upon either the flexibly mounted stator (FMS) or flexibly mounted rotor (FMR) configurations (Metcalfe, 1981). Either of these configurations is suitable for sealing a single shaft which penetrates a stationary housing. Green (1989 and 1990) showed that the dynamic response of the FMR configuration is superior to that of the FMS (Green and Etsion, 1985) both in terms of stability and steady-state misalignment. The gyroscopic moments, which result from the rotation of the flexibly mounted rotor, tend to counteract the misalignments resulting from manufacturing tolerances, resulting in seal faces which remain more parallel.

If it is necessary to seal between two rotating shafts rather than between a shaft and a housing, the seal will of necessity contain two rotors and no stator (Fig. 1). Such a seal was proposed by Miner (1992) for a gas turbine engine application. The performance advantage of the FMR configuration over the FMS provokes interest in whether a further gain can be obtained by flexibly mounting both rotors in a two-rotor seal. Wileman and Green (1991) have described the configuration in which both elements are flexibly mounted and attached to rotating shafts, denoting it FMRR. If one of the shaft speeds is zero, then the seal will have both a flexibly mounted rotor and a flexibly mounted stator, and this degenerate case is denoted FMSR.

Each of the rotors in a concentric FMRR seal possesses three degrees of freedom: an axial translation and two tilts about perpendicular axes located in the plane of the faces. The dynamic responses of the two elements are coupled by the fluid film, which transmits axial forces and tilting moments. Green (1990) showed that the axial vibrations in an FMR seal are decoupled from the angular modes, so that the solutions for the axial modes can be obtained independently. Wileman and Green (1991) showed that this decoupling extends to the FMRR configuration, so that its axial vibrations can be obtained by a trivial extension of the FMR result. This work, therefore, focuses exclusively on the response in the angular degrees of freedom. The equations of motion are derived for the four angular degrees of freedom based upon the rotor dynamic coefficients derived by Wileman and Green (1991) and including the inertia properties of the rotors and the properties of the flexible supports. The forcing functions in the system are seen to result from the initial misalignment of the flexible supports. An analytical method is presented for obtaining the general steady-state response to these forcing functions. The steady-state response determines the leakage rate of the seal and whether face contact occurs as a result of excessive steady-state runout.

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Kinematic Description

Wileman and Green (1991) described a kinematic model for the concentric FMRR seal configuration (Figs. 2 and 3). Figure 2 shows the inertial reference frame, $\xi\eta\zeta$, and the principal systems for both elements, denoted $x_n y_n z_n$. Throughout the analysis, n is used to denote the element number, either 1 or 2, and these numbers can be assigned arbitrarily provided they are applied consistently throughout the analysis. Figure 3 illustrates the various coordinate systems as viewed along the direction of the system centerline. This vector diagram shows the relationships between the inertial, principal, and fluid film systems described below.

The inertial moments acting upon the rotors depend upon their absolute motions, and are expressed in the inertial system $\xi\eta\zeta$ (Fig. 2). It is also with respect to this system that the rotations of the other reference frames are measured. The equations of motion for the system are obtained by transforming the flexible support and fluid film moments into this inertial frame, then forming a moment balance between these applied moments and the inertial moments.

A shaft-fixed reference frame $X_n Y_n Z_n$ is associated with each of the shafts, and rotates with the shaft, so that the axis Z_n is parallel to axis ζ, and X_n corresponds to ξ at $t = 0$ and leads ξ by the precession angle $\omega_n t$ thereafter.

The moments applied by the flexible support depend upon the motion of each element with respect to the shaft upon which it is mounted and are described using the principal coordinate systems $x_n y_n z_n$. These reference frames are defined such that the x axis is always the nutation axis of the seal element with respect to the shaft, and the y axis indicates the direction of maximum deviation of the face from the aligned position. The misalignments are denoted γ_1 and γ_2, where, as usual, the subscript denotes the element number. The precession angle ψ_{an} is defined as the angle by which the x_n axis leads the ξ axis at any instant.

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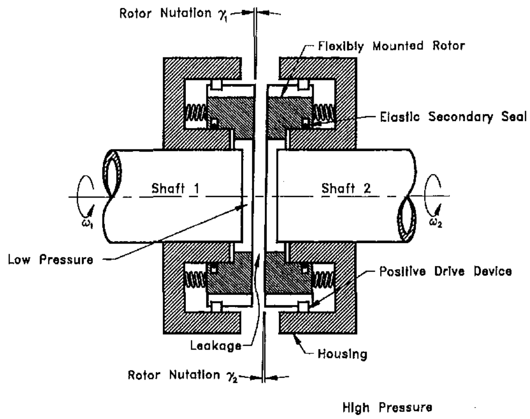

Fig. 1 Schematic of an FMRR mechanical seal

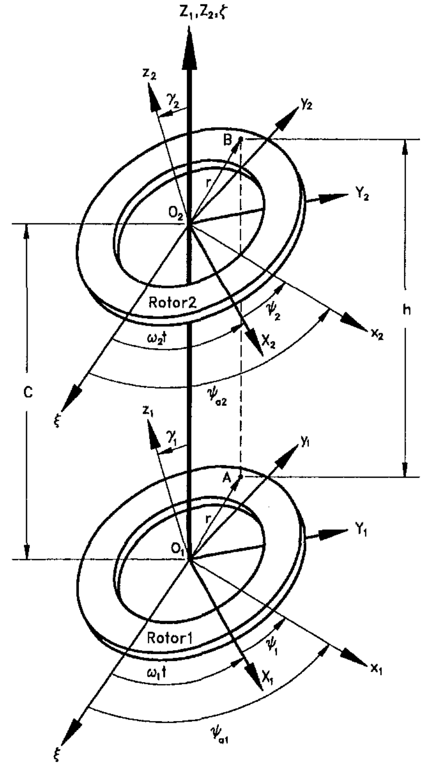

Fig. 2 FMRR seal kinematic model and coordinate systems

The moments applied by the fluid film depend upon the positions and velocities of the two elements with respect to one another. The relative position between the elements is expressed using the fluid film coordinate system. This rectangular system consists of the axes 123 (Fig. 3) which rotate such that the relative misalignment of the two elements, denoted by γ, is measured about axis 1 while axis 2 always corresponds to the direction of maximum film thickness (Wileman and Green, 1991).

The tilt angles in a mechanical seal are very small, typically the order of a milliradian. These small angles can be treated as vectors and can be resolved from one reference frame into another. Further, small angle approximations can be used for trigonometric functions of the tilts. Noting these assumptions, the relative tilt between the elements, γ, can be defined as the vector difference of the absolute tilts of the two elements. Thus, from Fig. 3,

$$\gamma = \gamma_2 \cos \phi_2 - \gamma_1 \cos \phi_1 \tag{1}$$

where ϕ_1 and ϕ_2 represent relative precession angles between the principal systems and the fluid film system.

Dynamic and Applied Moments

Once all of the moments have been resolved into components in the inertial system, moment balances are performed about both the ξ and η axes for each of the two elements. The four equations of motion which result are normalized using the procedure described by Wileman and Green (1991). The definitions of the normalized variables are repeated in the nomenclature of this work for reference. The moments representing the

Nomenclature

- c_n = inertia ratio of element n, I_n/J_n
- C = instantaneous seal centerline clearance
- C_0 = equilibrium centerline clearance
- d_{11} = fluid film damping coefficient, $2\pi R_m^3 G_0$
- d_{s11}^* = support damping coefficient, angular mode
- d_{s11} = dimensionless damping coefficient, $d^* \omega_{\text{ref}} C_0 / S r_o^4$
- E_0 = stiffness parameter, $(1 - R_i) R_m / 2 + \beta (1 - R_i)$
- F_n^* = axial force
- F_n = normalized axial force, $F^* / S r_o^2$
- G_0 = damping parameter, $\{\ln [1 + \beta(1 - R_i)] - 2\beta(1 - R_i)/[2 + \beta(1 - R_i)]\} / \beta^3 (1 - R_i)^2$
- h = film thickness
- H = normalized film thickness, h/C
- I^* = dimensional transverse moment of inertia
- I = normalized transverse moment of inertia, $I^* \omega_{\text{ref}}^2 C_0 / S r_o^4$
- J^* = dimensional polar moment of inertia
- J = normalized polar moment of inertia, $J^* \omega_{\text{ref}}^2 C_0 / S r_o^4$
- k_{11} = fluid film direct stiffness coefficient, $\pi (P_o - P_i)(\beta R_i - 1) E_0^2$
- k_{12} = fluid film cross-coupled stiffness coefficient, $d_{11}[\dot{\psi}_{ff} - (\omega_1 + \omega_2)/2]$
- k_{sn}^* = support stiffness coefficient, element n
- k = dimensionless stiffness coefficient, $k^* C_0 / S r_o^4$
- M_n^* = moment applied to element n
- M_n = normalized moment, $M_n^* / S r_0^3$
- p = pressure
- P = normalized pressure, p/S
- r = radius
- R = normalized radius, r/r_o
- S = seal parameter, $6 \mu \omega_{\text{ref}} (r_o/C_0)^2 (1 - R_i)^2$
- t^* = time
- t = normalized time, $\omega_2 t^*$
- β^* = coning angle
- β = normalized coning angle, $\beta^* r_o / C_0$
- γ^* = relative tilt angle (radians)
- γ_n^* = tilt angle of element n (radians)
- γ = normalized tilt, $\gamma^* r_o / C_0$
- μ = viscosity
- ϕ_n = phase angle between element principal and fluid film systems
- ψ_{ff} = absolute precession of fluid film system
- ψ_n = relative precession angle
- ψ_{an} = absolute precession angle
- ω_n^* = shaft angular speed of Element n
- ω_n = normalized shaft speed, $\omega_n^* / \omega_{\text{ref}}$
- ω_{ref} = reference shaft speed (used for normalization)
- ω_a = mean shaft speed

Subscripts

- 0 = equilibrium value
- i = inner radius
- m = mean radius
- n = element number (n = 1 or 2)
- o = outer radius

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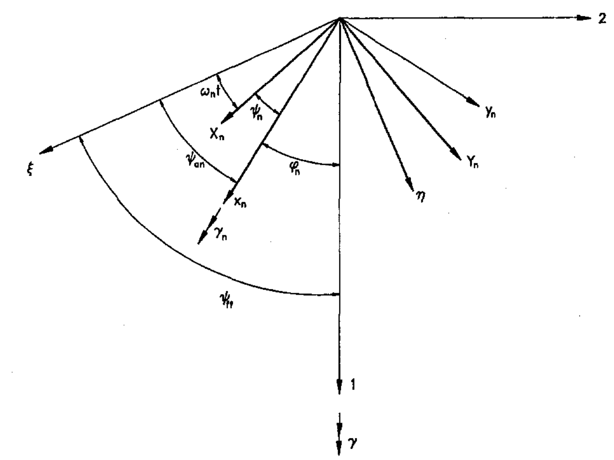

Fig. 3 Vector diagram showing relationship of coordinate axes for a single element as viewed along the system centerline

inertia effects are of course based upon the motion of the elements relative to the inertial system. The applied moments in the system have two sources: the flexible support and the fluid film.

Green (1990) derived the dynamic moments for the FMR configuration in the inertial reference frame. These moments are the same for the FMRR configuration, and for element n their normalized form is

$$M_{\xi nd} = I_n \ddot{\gamma}_{\xi n} + J_n \omega_n \dot{\gamma}_{\eta n}$$
$$M_{\eta nd} = I_n \ddot{\gamma}_{\eta n} - J_n \omega_n \dot{\gamma}_{\xi n} \qquad (2)$$

where the d in the subscript indicates that these are dynamic moments and the second term in each expression represents a gyroscopic moment.

The flexible support of each element usually consists of either a metal bellows or a combination of springs and an O-ring. If the properties of a flexible support are axisymmetric, then the moments it applies depend only upon the tilt of the element relative to the support.

Expressed in the principal system of the element, the normalized support moments for element n are (Wileman, 1994)

$$M_{xns} = -k_{sn}(\gamma_n - \gamma_{ni}\cos\psi_n) - d_{sn}\dot{\gamma}_n$$
$$M_{yns} = -d_{sn}\dot{\psi}_n\gamma_n - k_{sn}\gamma_{ni}\sin\psi_n \qquad (3)$$

where the s in the subscripts indicates that these are support effects, and k_{sn} and d_{sn} are the angular stiffness and damping coefficients for the support of element n. Green and Etsion (1986b) have presented a method for empirically estimating the values of k_s and d_s for supports consisting of elastomer O-rings. The terms in (3) containing γ_{ni}, the initial misalignment of the support, will be shown to produce the harmonic forcing functions in the system. These initial support misalignments represent the deviation of the supports from the perfectly aligned position before assembly and normally result from imperfections in manufacturing.

The support moments in (3) are resolved into the inertial system using (Fig. 3)

$$\begin{Bmatrix} M_\xi \\ M_\eta \end{Bmatrix}_n = \begin{pmatrix} \cos\psi_{an} & -\sin\psi_{an} \\ \sin\psi_{an} & \cos\psi_{an} \end{pmatrix} \begin{Bmatrix} M_x \\ M_y \end{Bmatrix}_n \qquad (4)$$

The tilts in the principal systems are related to those in the inertial system using

$$\gamma_{\xi n} = \gamma_n\cos\psi_{an}; \quad \gamma_{\eta n} = \gamma_n\sin\psi_{an} \qquad (5)$$

The components in the inertial system are obtained by substituting (3) into (4) and simplifying using (5) with the result

$$M_{\xi ns} = -k_{sn}\gamma_{\xi n} - d_{sn}(\dot{\gamma}_{\xi n} + \omega_n\gamma_{\eta n})$$
$$M_{\eta ns} = -k_{sn}\gamma_{\eta n} - d_{sn}(\dot{\gamma}_{\eta n} - \omega_n\gamma_{\xi n}) \qquad (6)$$

Note that the support stiffness and damping coefficients will generally be different for the two elements.

Wileman and Green (1991) have obtained rotor dynamic coefficients which allow the fluid behavior to be introduced directly into the equations of motion for the FMRR system. These coefficients express the changes in the moments which the fluid film applies to the element faces as a result of small deviations from an equilibrium position.

The rotor dynamic coefficients are derived in the fluid film reference frame, 123, because the film geometry is symmetric in this system. In terms of the coefficients, the fluid film moments applied to element 2 are

$$M_1 = -k_{11}\gamma - d_{11}\dot{\gamma} \qquad (7a)$$

$$M_2 = -k_{12}\gamma = -d_{11}\left[\dot{\psi}_{ff} - \tfrac{1}{2}(\omega_1 + \omega_2)\right]\gamma \qquad (7b)$$

where the subscript for each moment denotes the axis in the fluid film reference frame about which the moment is applied. The normalized rotor dynamic coefficients obtained by Wileman and Green (1991) are provided in the nomenclature for reference. These coefficients are valid for incompressible, isoviscous fluid films in which the hydrostatic pressure is sufficient to suppress cavitation.

The moments in the fluid film system, (7), are transformed into the element principal systems using (Fig. 3)

$$\begin{Bmatrix} M_x \\ M_y \end{Bmatrix} = \begin{pmatrix} \cos\phi_2 & -\sin\phi_2 \\ \sin\phi_2 & \cos\phi_2 \end{pmatrix} \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} \qquad (8)$$

The resulting moments are in turn resolved into the inertial system using (4) and simplified using (5) and standard trigonometric identities.

When all of the moments are expressed in the inertial reference, the equations of motion are obtained by equating the applied moments to the dynamic moments. To simplify the equations, a few shorthand terms are defined:

$$k_n = k_{11} + k_{sn}; \quad d_n = d_{11} + d_{sn}$$
$$D_n = \tfrac{1}{2}d_{11}(\omega_1 + \omega_2) + \omega_n d_{sn}$$
$$D_{11} = \tfrac{1}{2}d_{11}(\omega_1 + \omega_2)$$

The resulting equations of motion are

$$I_2\ddot{\gamma}_{\xi 2} + d_2\dot{\gamma}_{\xi 2} - d_{11}\dot{\gamma}_{\xi 1} + J_2\omega_2\dot{\gamma}_{\eta 2} + k_2\gamma_{\xi 2} - k_{11}\gamma_{\xi 1} + D_2\gamma_{\eta 2} - D_{11}\gamma_{\eta 1} = k_{s2}\gamma_{2i}\cos\omega_2 t$$
$$I_2\ddot{\gamma}_{\eta 2} + d_2\dot{\gamma}_{\eta 2} - d_{11}\dot{\gamma}_{\eta 1} - J_2\omega_2\dot{\gamma}_{\xi 2} + k_2\gamma_{\eta 2} - k_{11}\gamma_{\eta 1} - D_2\gamma_{\xi 2} + D_{11}\gamma_{\xi 1} = k_{s2}\gamma_{2i}\sin\omega_2 t$$
$$I_1\ddot{\gamma}_{\xi 1} + d_1\dot{\gamma}_{\xi 1} - d_{11}\dot{\gamma}_{\xi 2} + J_1\omega_1\dot{\gamma}_{\eta 1} + k_1\gamma_{\xi 1} - k_{11}\gamma_{\xi 2} + D_1\gamma_{\eta 1} - D_{11}\gamma_{\eta 2} = k_{s1}\gamma_{1i}\cos\omega_1 t$$
$$I_1\ddot{\gamma}_{\eta 1} + d_1\dot{\gamma}_{\eta 1} - d_{11}\dot{\gamma}_{\eta 2} - J_1\omega_1\dot{\gamma}_{\xi 1} + k_1\gamma_{\eta 1} - k_{11}\gamma_{\eta 2} - D_1\gamma_{\xi 1} + D_{11}\gamma_{\xi 2} = k_{s1}\gamma_{1i}\sin\omega_1 t \qquad (9)$$

Note that the equations of motion in this form are linear with respect to the kinematic variables in the inertial reference frame, but that the equations are coupled by the fluid film and gyroscopic effects. The first two of these equations represent moment balances for element 2 about the ξ and η axes, respectively, and the forcing function for each of the two equations results from the initial misalignment of the support of element 2, γ_{2i}. The third and fourth equation represent similar moment bal-

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ances for element 1, with the forcing functions resulting from the misalignment of the support of element 1, γ_{1i}.

Steady-State Analysis

As stated previously, the equations of motion (9) expressed in the inertial reference frame are linear and contain harmonic forcing functions which result from the initial misalignments of the flexible supports. The response of the system of equations to these two forcing functions represents the steady-state response of the seal to the initial misalignments. In this inertial system, each of the forcing functions is harmonic with a frequency equal to the rotation speed of the associated shaft. Because the system of equations is linear, the response of the system to each forcing function can be obtained separately, and the two can be combined using superposition to obtain the total response.

Since the initial misalignment associated with each of the supports rotates with the speed of the shaft to which the support is attached, the forcing function, and therefore the response, will be constant in the coordinate system which rotates with the same shaft. If the equations of motion are transformed into this system, the equations remain linear but the terms containing time derivatives vanish.

Transformation of Eqs. (9) into the shaft-fixed systems is accomplished by resolving all of the moments about the inertial axes into components about the axes which rotate with the shaft. To obtain the total response of the system it is necessary to perform this process twice. The equations are transformed to the shaft 2 system to determine the response of both elements to the initial misalignment in the support of element 2. Then, similarly, the equations are transformed into the shaft 1 system. The responses to the two forcing functions are then superposed to determine the maximum value of the combined response.

In this section the equations of motion will be transferred to the shaft-fixed system of element 2. Hence, we require that $\gamma_{1i} = 0$. Note that each of the equations of motion in the inertial system represents a moment balance about one of the inertial axes. These moments can be transformed from the inertial system into the shaft-fixed system by noting (Fig. 3)

$$\begin{Bmatrix} M_I \\ M_J \end{Bmatrix} = \begin{pmatrix} \cos \omega_n t & \sin \omega_n t \\ -\sin \omega_n t & \cos \omega_n t \end{pmatrix} \begin{Bmatrix} M_\xi \\ M_\eta \end{Bmatrix} \qquad (10)$$

where M_I and M_J represent moments about shaft-fixed axes. The tilts about the inertial axes are resolved into the shaft-fixed system in a similar manner, leading to the definitions

$$\gamma_{2I} = \gamma_\xi \cos \omega_2 t + \gamma_\eta \sin \omega_2 t$$

$$\gamma_{2J} = -\gamma_\xi \sin \omega_2 t + \gamma_\eta \cos \omega_2 t \qquad (11)$$

where γ_{2I} and γ_{2J} represent the tilts of element 2 resolved into components in the shaft-fixed system of element 2. Note that γ_{1I} and γ_{1J} are resolved onto the same axes, but represent the tilts of element 1.

To obtain the equations of motion in the new system, we substitute the first two equations in (9) for M_ξ and M_η in (10). Then substitute the definitions of (11) to obtain the equations of motion for element 2 in the shaft-fixed system. To obtain the equations for element 1, a similar procedure is applied to the third and fourth equations of (9).

The result is four equations of motion as before, but expressed in the shaft-fixed system. For the system which rotates with the shaft of rotor 2, these equations are

$$I_2\ddot{\gamma}_{2I} + (d_{11} + d_{s2})\dot{\gamma}_{2I} + (J_2 - 2I_2)\omega_2\dot{\gamma}_{2J} + [(J_2 - I_2)\omega_2^2 + k_{11} + k_{s2}]\gamma_{2I} - \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)\gamma_{2J} - d_{11}\dot{\gamma}_{1I} - k_{11}\gamma_{1I} + \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)\gamma_{1J} = k_{s2}\gamma_{2i} \qquad (12a)$$

$$I_2\ddot{\gamma}_{2J} - (J_2 - 2I_2)\omega_2\dot{\gamma}_{2I} + (d_{11} + d_{s2})\dot{\gamma}_{2J} + \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)\gamma_{2I} + [(J_2 - I_2)\omega_2^2 + k_{11} + k_{s2}]\gamma_{2J} - d_{11}\dot{\gamma}_{1J} - \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)\gamma_{1I} - k_{11}\gamma_{1J} = 0 \qquad (12b)$$

$$I_1\ddot{\gamma}_{1I} + (d_{11} + d_{s1})\dot{\gamma}_{1I} + (J_1\omega_1 - 2I_1\omega_2)\dot{\gamma}_{1J} + [(J_1\omega_1 - I_1\omega_2)\omega_2 + k_{11} + k_{s1}]\gamma_{1I} + [d_{s1}(\omega_2 - \omega_1) - \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)]\gamma_{1J} - d_{11}\dot{\gamma}_{2I} - k_{11}\gamma_{2I} + \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)\gamma_{2J} = 0 \qquad (12c)$$

$$I_1\ddot{\gamma}_{1J} - (J_1\omega_1 - 2I_1\omega_2)\dot{\gamma}_{1I} + (d_{11} + d_{s1})\dot{\gamma}_{1J} - [d_{s1}(\omega_2 - \omega_1) - \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)]\gamma_{1I} + [(J_1\omega_1 - I_1\omega_2)\omega_2 + k_{11} + k_{s1}]\gamma_{1J} - d_{11}\dot{\gamma}_{2J} - \tfrac{1}{2} d_{11}(\omega_2 - \omega_1)\gamma_{2I} - k_{11}\gamma_{2J} = 0 \qquad (12d)$$

The only forcing function in the equations of motion above is the constant function $k_{s2}\gamma_{2i}$. Because the equations are linear, the response of the system to such a forcing function must also be constant. Thus, the time derivatives of the nutation variables will vanish at steady-state. If these time derivatives are removed from (12), the equations can be written in matrix form as

$$(A_2) \begin{Bmatrix} \gamma_{2I} \\ \gamma_{2J} \\ \gamma_{1I} \\ \gamma_{1J} \end{Bmatrix} = \begin{Bmatrix} k_{s2}\gamma_{2i} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (13)$$

where

$$\mathbf{A}_2 = \begin{pmatrix} (J_2 - I_2)\omega_2^2 + k_{11} + k_{s2} & -\dfrac{1}{2}d_{11}(\omega_2 - \omega_1) & -k_{11} & \dfrac{1}{2}d_{11}(\omega_2 - \omega_1) \\ \dfrac{1}{2}d_{11}(\omega_2 - \omega_1) & (J_2 - I_2)\omega_2^2 + k_{11} + k_{s2} & -\dfrac{1}{2}d_{11}(\omega_2 - \omega_1) & -k_{11} \\ -k_{11} & \dfrac{1}{2}d_{11}(\omega_2 - \omega_1) & (J_1\omega_1 - I_1\omega_2)\omega_2 + k_{11}k_{s1} & \left(d_{s1} - \dfrac{d_{11}}{2}\right)(\omega_2 - \omega_1) \\ -\dfrac{1}{2}d_{11}(\omega_2 - \omega_1) & -k_{11} + k_s & -\left(d_{s1} - \dfrac{d_{11}}{2}\right)(\omega_2 - \omega_1) & (J_1\omega_1 - I_1\omega_2)\omega_2 + k_{11} + k_{s1} \end{pmatrix}$$

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The solution to (13) can be obtained by inverting the matrix $\mathbf{A}_2$. If the seal design and operating conditions are known, the matrix will consist of numerical values and can be easily inverted, allowing (13) to be solved for the steady-state tilts.

It is also possible to supply values for all but one or two of the variables in the matrix before inverting it. The resulting solution can then be presented as a plot of the response versus these variables, allowing the seal design to be investigated parametrically.

Response to Initial Misalignment of Element 1

The response of the system to the initial misalignment of element 1 is obtained in a manner similar to the preceding analysis. The equations are transformed to the reference frame attached to shaft 1, and the initial misalignment of element 1 appears as a constant forcing function in the system while the initial misalignment of element 2 is assumed to be zero. Then, following a development similar to that above, a set of equations in the new system is obtained with a forcing function which is again constant.

Eliminating the time derivatives as before, the equations at steady-state in matrix form are

$$(\mathbf{A}_1)\begin{Bmatrix}\gamma_{2I}\\ \gamma_{2J}\\ \gamma_{1I}\\ \gamma_{1J}\end{Bmatrix} = \begin{Bmatrix}0\\ 0\\ k_{s1}\gamma_{1i}\\ 0\end{Bmatrix} \qquad (14)$$

where

$$\mathbf{A}_1 = \begin{pmatrix} (J_2\omega_2 - I_2\omega_1)\omega_1 + k_{11} + k_{s2} & \left(d_{s2} + \frac{d_{11}}{2}\right)(\omega_2 - \omega_1) & -k_{11} & -\frac{1}{2}d_{11}(\omega_2 - \omega_1) \\ -\left(d_{s2} + \frac{d_{11}}{2}\right)(\omega_2 - \omega_1) & (J_2\omega_2 - I_2\omega_1)\omega_1 + k_{11} + k_{s2} & +\frac{1}{2}d_{11}(\omega_2 - \omega_1) & -k_{11} \\ -k_{11} & -\frac{1}{2}d_{11}(\omega_2 - \omega_1) & (J_1 - I_1)\omega_1^2 + k_{11} + k_{s1} & \frac{1}{2}d_{11}(\omega_2 - \omega_1) \\ \frac{1}{2}d_{11}(\omega_2 - \omega_1) & -k_{11} & -\frac{1}{2}d_{11}(\omega_2 - \omega_1) & (J_1 - I_1)\omega_1^2 + k_{11} + k_{s1} \end{pmatrix}$$

The Relative Misalignment

The preceding analysis provides the magnitudes of the rotating responses to each of the initial misalignments. Each of the responses represents a constant tilt magnitude which precesses at the speed of the associated forcing function. The tilts γ_I and γ_J represent components of the misalignment of each element with respect to the shaft to which it is attached.

The relative misalignment between the two elements, however, is a more important measure of the seal performance. It is this value which determines the leakage rate of the seal, and the maximum relative misalignment determines whether the two seal faces come into contact.

For each forcing function, the relative misalignment is computed as the vector difference defined in (1). The total relative misalignment in the system is obtained as a vector sum of the individual relative misalignments of the two forcing functions. This total misalignment will actually be quite complex, a result of summing vectors which are precessing at different speeds, and possibly in different directions. The minimum relative misalignment will occur when the individual relative tilts are parallel and in the same sense, and the maximum relative misalignment will occur when the tilts are parallel with opposing senses (i.e., with directions differing by π). The maximum relative tilt will simply be the sum of the magnitudes of the two individual relative tilts. The instantaneous relative tilt will be a periodic function bounded by the maximum and minimum relative misalignments.

Conclusions

The equations of motion for a general FMRR seal were derived in both inertial and principal coordinate systems. The equations describe the dynamic behavior of the angular degrees of freedom of the rotors, and they include the effects of rotor inertia, the fluid film stiffness and damping, and the properties of the flexible support. The equations in the inertial system are linear with harmonic forcing functions resulting from the initial support misalignment and with frequencies equal to the shaft rotation speeds.

To obtain an analytical solution for the steady-state response, the equations of motion were resolved into components in reference frames fixed to the rotating shafts. The resulting equations are linear with constant forcing functions, so that the steady-state solution is readily obtained. The analysis presented in this paper provides an analytical tool for evaluating the steady-state behavior of any general FMRR or FMSR seal.

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