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Letter to the editor

Discussion on: Jia Ma, Menghao Bai, Jie Wang, Shuai Dong, Hao Jie, Bo Hu, Lairong Yin, "A novel variable restitution coefficient model for sphere–substrate elastoplastic contact/impact process," Mechanism and Machine Theory 202 (2024)☆

The readers ought to be aware of some unfounded and deficient theories in the subject paper [1]. The objective here is to highlight the said deficiencies, and to recast the proper foundation for the contact mechanics that is central to the study of the coefficient of restitution.

Starting with Eq. (1), it is simplistic and inappropriate to handle a three-dimensional (3D) Hertzian stress-state [2]. Moreover, Eq. (1) makes no distinction for cases of contact or impact that transpire between dissimilar materials. The authors likewise failed to review highly relevant literature, old and recent, and they have not compared their results with published experimental and numerical results on the coefficient of restitution.

It is well established that spherical Hertzian contact induces a 3D stress state. The reader would be better served by reviewing the additional Ref. [2] as listed in the additional references given below (Ref. [2] was published nearly two decades ago). The following remarks encapsulate the underlying fundamentals. In this discussion S and σ signify the strength and stress, respectively. Reference [2] derives *analytically* all of the relevant parameters at the onset of yielding (i.e., plasticity) for hemispherical and cylindrical contacts. It *specifically analyzes* contacting bodies that have *dissimilar material properties.* Attention should also be made to whether the materials are *ductile or brittle* (an issue that is totally ignored by the authors in the subject paper). Specifically, when two contacting bodies are ductile and have dissimilar material properties – say, $S_{y1} \neq S_{y2}$, etc., – it is *imperative* to use the procedure taught by Green [2] to calculate the critical parameters (significant errors might ensue otherwise). Briefly, in normal elastic contacts, Green [2] defined $C=p_o/\sigma_{e\text{-max}}$ as the ratio between the maximum Hertzian contact pressure, p_o, and the maximum von Mises stress, $\sigma_{e\text{-max}}$ (which transpires in 3D contact stress state *under* the surface along the contacting bodies' centerline). *That ratio solely dependents upon the Poisson ratio,* see expressly [2–3], $C(\nu)=1.295\exp(0.736\nu)$. In Hertzian contacts the two bodies share the same contact area and the same maximum contact pressure, p_o. By abiding to the von Mises criterion at yielding onset, $\sigma_{e\text{-max}} = S_y$, the product $C(\nu)\bullet S_y$ produces *identically* the corresponding *critical contact pressure,* $p_{oy} \equiv C(\nu)\bullet S_y$, that would initiate yielding *for that particular body*. For *dissimilar materials*, Green [2] instructs to pick the *smallest* between two *maximum contact pressure possibilities* to decide which of the contacting bodies yields first. Hence,

$$\left(p_{oy}\right)_{\min} \equiv \left(CS_y\right) = \min\left[C(\nu_1)S_{y1}, C(\nu_2)S_{y2}\right] \tag{1}$$

where,

$$C(\nu) = 1.295\exp(0.736\nu) \tag{2}$$

These two equations, (1) and (2), are now used to calculate the critical interference, load, and contact radius, which are also derived in [2]. For spherical contact the corresponding *critical* displacement, force, and radius are given by:

$$\delta_y = \left(\frac{\pi\left(p_{oy}\right)_{\min}}{2E^*}\right)^2 R = \left(\frac{\pi\left(CS_y\right)}{2E^*}\right)^2 R \tag{3}$$

$$F_y = \frac{1}{6}\left(\frac{R}{E^*}\right)^2\left[\pi\left(p_{oy}\right)_{\min}\right]^3 = \frac{1}{6}\left(\frac{R}{E^*}\right)^2\left[\pi\left(CS_y\right)\right]^3 \tag{4}$$

☆ **A discussion by** Itzhak Green **on: Jia Ma, Menghao Bai, Jie Wang, Shuai Dong, Hao Jie, Bo Hu, Lairong Yin,** "A novel variable restitution coefficient model for sphere–substrate elastoplastic contact/impact process," Mechanism and Machine Theory 202 (2024), https://doi.org/10.1016/j.mechmachtheory.2024.105773.

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$$a_y = \frac{\pi\left(p_{oy}\right)_{\min}}{2E^*}R = \frac{\pi\left(CS_y\right)}{2E^*}R \tag{5}$$

where R is the composite radius calculated by $1/R = 1/R_1+1/R_2$, and $E^* = [(1\text{-}\nu_1^2)/E_1 + (1\text{-}\nu_2^2)/E_2]^{-1}$ is the equivalent modulus of elasticity. Emphasizing, C and S_y always appear grouped together in a product form (CS_y) to signify the *minimum* value of the *maximum contact pressure* that brings upon yielding onset, $(p_{oy})_{\min} \equiv (CS_y)$, according to Eq. (1) above, which accounts for dissimilar material properties and radii as neceessary.

Another issue is with the authors' usage of the ratio given in their Eq. (3), $\varepsilon_y^* = S_y / E^*$. Again, as discussed in [3,5] for dissimilar materials, and for the same reasons given above for Eq. (1), one must apply:

$$\varepsilon_y^* = \begin{cases} S_{y1}/E^* & \leftrightarrow \quad C(\nu_1)S_{y1} \leq C(\nu_2)S_{y2} \\ S_{y2}/E^* & \leftrightarrow \quad C(\nu_1)S_{y1} > C(\nu_2)S_{y2} \end{cases} \tag{6}$$

The current Eqs. (1-6) given herein should replace the corresponding equations in Ref. [1], which are likely to affect the results reported in the subject paper.

Then there is even a broader issue of ignoring highly relevant literature of old (Zener, [4]), and recent (Green, [5]). The authors fail to compare results with other published *experimental* and *FEA results* of spheres impacting finite thickness plates as reported by Higgs et al. [6–8]. The work in Ref. [5] provides a closed-form expression (see Eq. (20) there) that predicts the coefficient of restitution with *high fidelity* as it matches *virtually perfectly* the said *FEA* and *experimental results* in [6–8]. Briefly, *elastoplastic impact* and *wave propagation* are fused into a single Eq. (20) in Ref. [5] as it predicts the coefficient of restitution for a whole array of cases of impacting spheres against flat plates where the bodies may have *vastly dissimilar material properties (including ductility and brittleness)*. It is counterproductive when building blocks such as the aforementioned (Refs. [2–8]) are bypassed in favor of simplistic and unfounded theories that could distort results and hinder future advancement of research and development.

CRediT authorship contribution statement

Itzhak Green: Writing – original draft.

Declaration of competing interest

The author declares that he has no conflict of interest.

Data availability

Data will be made available on request.

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