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AN INVESTIGATION OF THREE DIMENSIONAL ELASTIC-PLASTIC HEMISPHERICAL SLIDING CONTACT, PART I: MODELING AND VALIDAITON

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ABSTRACT

This work presents a three dimensional (3D) finite element analysis (FEA) of an elastic-plastic hemispherical contact model for two hemispherical bodies sliding across each other with various preset vertical interferences. The boundary conditions, model simplifications, and the normalization scheme are presented. Sample results from this FEA investigation are compared to a semi-analytical solution to validate the methodology.

INTRODUCTION

An investigation of elastic-plastic sliding asperity contact using a three dimensional (3D) finite element analysis (FEA) is presented in this part. Sliding contact is an important phenomenon in both the macro and micro scales. In the macro scale, it is important to consider friction, wear, and residual deformation that result from contact situations such as in rolling element bearings. In the micro scale, it is known that nominally smooth surfaces do indeed have undulations in their surface profile and the real area of contact is just a small percentage of the nominal area of contact. These high points, or asperities, are known to deform plastically during sliding. Thus, it is important to know how the deformed geometry, residual stresses, and surface condition affect the sliding process. This information for a pair of asperities provides the kernel of the solution for any rough surface described statistically. The model presented here has been normalized in order to apply the results to both macro and micro scale geometries.

Though work has been done in the area of surface contact, in most cases either simplifying assumptions have ignored important phenomena or less than satisfactory results have been produced. There have been many works, mainly based on Green [1, 2], which analyzed friction and adhesion of triangular shaped contact interfaces, that have analyzed fully plastic contact interfaces. In reality though, the contact junctions should realistically be modeled as spherical in shape.

Historically, work in the area of hemispherical contact has been done in a sphere-on-flat configuration. Faulkner and Arnell [3] present the first work that models sphere-on-sphere contact using an FEA approach. Very few useful results are presented in this work and the method resulted in extremely long execution times (over 960 hours). The objective of this work is to present a generalized modeling method that can be applied to many material combinations.

MODELING METHOD

A schematic representation of the sliding process is shown in Figure 1, lying in the *xy* plane. Axis *x* is along the direction of sliding, axis *y* is vertical, and axis *z* is normal to the said plane *xy*. In this analysis a displacement, Δx is applied to the top sphere while the bottom sphere is held stationary. The horizontal displacement imposed to complete sliding increases with increasing the vertical interference, ω. As the vertical interference increases, more plastic deformation occurs and residual stresses and strains are present upon disengagement. In the model, the top sphere is initially placed just in contact with the lower sphere and then slides across the bottom sphere until the spheres are no longer in contact.

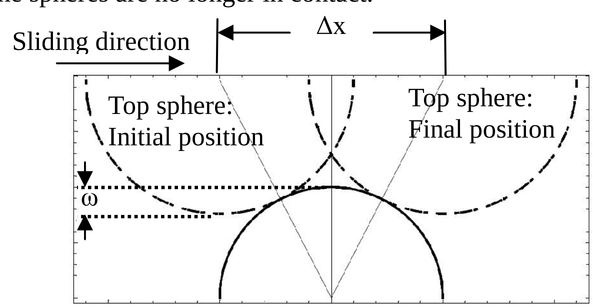

Figure 1: A schematic of the sliding process

This analysis is done using ABAQUS, a commercial FEA software package using linear brick (8-node) elements. In order to take advantage of the symmetry of the problem, each sphere is cut in half along the vertical *xy* plane and no displacement is allowed normal to this cut plane. This can be thought of as a roller boundary condition. Also, an assumption is made, and later confirmed, that under the interferences considered here there is little to no stress or deformation in areas far from the contact region (half-space assumption). This assumption is reasonable if one considers the fact that the contact half-width is much smaller than the radius of the sphere and as such, the stress distribution near the contact region is unaffected by the conditions far away from it. To take advantage of this each sphere is cut in half in the horizontal plane and once again a roller boundary condition is imposed on the top surface of the top sphere while vertical displacement is completely constrained on the bottom surface of the bottom sphere. The end result is a quarter-sphere model for each spherical body.

This analysis considers steel-on-steel and aluminum-on-copper contact. The material is modeled as elastic-perfectly plastic, but in order to help convergence, a 2% strain hardening based on the Young's modulus was used on the higher interference cases. This small amount of strain hardening has been shown to not significantly affect the results yet drastically improves upon execution time [4].

In the elastic domain and up to the onset of plasticity, the Hertzian solution [5] is used to obtain critical values of load, contact half-width, and strain energy [6]. As explained by Jackson et al [7], hardness is not implemented as a unique material property as it varies with the deformation itself as well as with other material properties such as yield strength, Poisson's ratio, and the elastic modulus. Instead, the critical vertical interference, ω_c, as derived by Green [6] for hemispherical contact, is employed. This quantity is derived by using the distortion energy yield criterion at the site of maximum von Mises stress by comparing the stress value with the yield strength, S_y. The critical values of force, P_c, contact area, A_c, and interference, ω_c, are:

$$P_c = \frac{(\pi C S_y)^3 R^2}{6E'^2} \; ; \; A_c = \frac{\pi^3 \left(C S_y R\right)^2}{\left(2E'\right)^2} \; ; \; \omega_c = \left(\frac{C S_y}{2E'}\right)^2 R \tag{1}$$

$$\frac{\sigma_e}{p_o} = \frac{1}{2}\sqrt{\frac{((1-2\nu-2\zeta^2(1+\nu)+2(\zeta+\zeta^3)(1+\nu)ArcCot\left[\zeta\right])^2}{(1+\zeta^2)^2}} \tag{2}$$

Where:

$$\frac{1}{R} = \frac{1}{R^1} + \frac{1}{R^2} \tag{3}$$

$$\frac{1}{E'} = \frac{1-\nu_1^{\,2}}{E_1} + \frac{1-\nu_2^{\,2}}{E_2} \tag{4}$$

$$\zeta_c = 0.38167 + 0.33136\nu \tag{5}$$

$$C = 1.30075 + 0.87825\nu + 0.54373\nu^2 \tag{6}$$

$$CS_y = \min(CS_{y1}, CS_{y2}) \tag{7}$$

The value of C is obtained from elasticity considerations, and the critical parameters are obtained at the point of yielding onset. The maximum elastic energy that can possibly be stored (up to the point of yielding onset) is used to normalize the net energy loss due to plastic deformation after sliding, and is given by Green [6] as:

$$U_c = \frac{\pi \left(C S_y\right)^5 R^3}{60E'^4} \tag{8}$$

Table 1 presents the material properties used in this analysis for steel-on-steel and aluminum-on-copper sliding contact. The critical values are calculated for both contact pairs and presented in Table 2. Since all the quantities are subsequently being normalized by the critical parameters in Eq. (1), the ensuing results apply for any geometry scale (and for as long as continuum prevails); therefore, the radii for the asperities in the FE model are subjectively (and conveniently) chosen to be $R_1 = R_2 = 1\ m$. It should be noted that the critical parameters are used solely for the purpose of normalization of the results, whether frictional or frictionless sliding.

Table 1: Material properties for the two spheres

Property	**Steel**	**Aluminum**	**Copper**
E	200 GPa	68.0 GPa	130 GPa
S_y	911.5 MPa	310 MPa	331 MPa
υ	0.32	0.326	0.33

Table 2: Critical values of parameters at the onset of plasticity for sliding between two spherical asperities

Parameter	**Steel-on-Steel**	**Al-on-Cu[1]**
CS_y	1.493 GPa	509.9 MPa
ω_c	0.2214 mm	0.1261 mm
P_c	346.1 kN	67.32 kN
A_c	347.8 mm^2	198 mm^2
U_c	30.65 J	3.395 J

1. Aluminum is yielding first

REPRESENTATIVE RESULTS AND VALIDATION

In order to validate the FEA method, the results are compared to a semi-analytical numerical method (SAM). For more information on the specifics of the methodology used here see Boucly et al [8]. Briefly, this semi-analytical method uses contact pressure on the surface that can be thought of as the summation of concentrated normal loads over the area of contact. Each of these concentrated loads has a corresponding influence on the displacement throughout the body. This influence is quantified using influence coefficients, which are actually the discretized form of Green's functions. The semi-analytical method takes advantage of this by using the superposition principle to sum at each location in the region of interest the displacements due to the contact pressure. Once this information is gathered the stresses, strains, and deformations can be calculated based on the material properties from the compatibility and equilibrium relations. An iterative process is used to incorporate the residual deformations present from a previous load step.

Figures 2 and 3 present a comparison of the horizontal and vertical reaction forces for the different vertical interferences for steel-on-steel frictionless sliding contact, respectively. As shown in the figures, the results are nearly identical for the smaller interference cases. As shown in Figure 2, with increasing preset interference, the semi-analytical results diverge from the FEA results once the spheres have passed the point of vertical alignment. The vertical reaction force curves, as shown in Figure 3, are also nearly identical for all the interference cases presented. The discrepancy between the FEA and SAM results at larger interferences may be due to the different implementation of material hardening: FEA uses a bilinear hardening (giving a larger force response in the loading phase), while SAM uses an elastic-perfectly plastic material. This comparison of results suggests that both the FEA and SAM adequately model the sliding contact phenomenon.

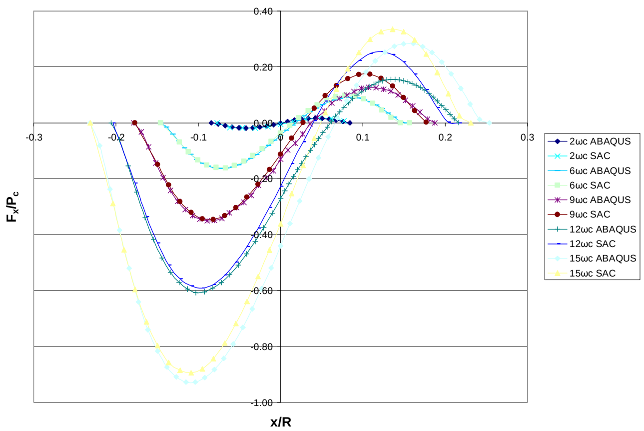

Figure 2: A comparison of the SAM and FEA results for the tugging force.

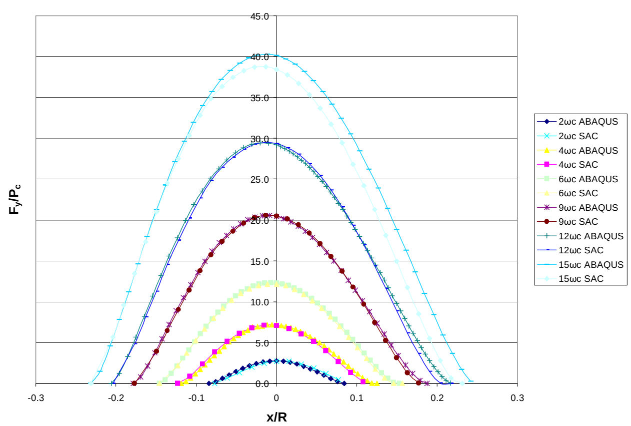

Figure 3: A comparison of the SAM and FEA results for the normal reaction force.

CONCLUSIONS

The main conclusions can be summarizes as followed:

1. A FEA modeling method is presented to analyze 3D hemispherical sliding contact.
2. A normalization scheme based on the distortion energy criterion is presented in order to apply the results to any scale.
3. Representative results are presented and validated against a semi-analytical technique. The FEA and SAM results agree rather well for all cases examined of frictionless sliding. Typically the SAM code runs in less than 10 percent of the FEA execution time.

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