Discussion of "Contact Unloading Behaviors of Elastic-Power-Law Strain Hardening Material Considering Indenter Elasticity Effect" (Chen, C., Wang, Q., Wang, H., Ding, H., Hu, W., Xie, W., Weng, P., Jiang, L., and Yin, X., 2022, ASME J. Tribol., 144(12), p. 121501)

Itzhak Green
School of Mechanical Engineering,
Georgia Institute of Technology,
Atlanta, GA 30332-0405
e-mails: green@gatech.edu; itzhak.green@me.gatech.edu

The readers ought to be aware of some claims based on unfounded theories in the subject paper [1], which could possibly distort results and hinder future research and development. To start, Eq. (3) in the subject paper is stated to be taken from Ref. [70] (Johnson's classical textbook). However, said Eq. (3) is imprecise because it uses a constant coefficient $\theta_y = 1.1$, as claimed. That coefficient does not account for the variability in compressibility of the materials in contact, i.e., the Poisson's ratios of the materials. The general range of Poisson's ratios, ν, for crystalline and engineering materials is between -1 and 0.5. For example, $\nu = 0.2$ for cast iron, $\nu = 0.18$ for glass, or $\nu = 0.44$ for gold. There are many materials and crystallines that have *negative* Poisson ratios (some are even time-dependent). Thus, limiting analyses and discussions to narrow subsets of Poisson ratios, say around 0.3, would likewise be narrow. That dependence upon Poisson's ratio is addressed not only in Ref. [70], but also in other dated work, such as by the Chang et al. (CEB) model [2]. The CEB derivation, however, has other flaws—see discussions in Refs. [3] and [4].

There is another, and even more significant problem in the subject paper in regards to what constitutes the "yield strength" when the half-space and the sphere in contact have dissimilar material properties. The authors base their Eq. (6) on references [40,71,72]. Starting with Ref. [40], in which no form of Eq. (6) is present, but the author there offers an "effective hardness," or a "reduced hardness" for bodies having dissimilar material properties. In one case it is postulated that $H = (1/H_1 + 2/H_2)^{-1}$, and then $H = (2/H_1 + 2/H_2)^{-1}$. Not only that these two equations yield very different results, more importantly, there is no scientific or engineering foundation for such a "reduced hardness." In Ref. [67], the same author in addition to the "reduced hardness" also offers the form of Eq. (6) used in the subject paper [1]. However, a review by Ghaednia et al. [5] states on these offerings: "Note that there appears to be no fundamental mechanics based derivation for [the said equations]… [it] is not possible for combining the plasticity properties of two contacting bodies [in that manner]." Then, Ref. [71] does not support anywhere the said Eq. (6). Reference [72] proposes *an ad hoc* equation (identical to Eq. (6)) but careful reading reveals that that postulation is particular to "Level III – plasticity," i.e., *full and complete* plasticity of the surfaces with particular attention directed at the *edges* of contact and *not* at the *onset* of yielding taking place *at a point under the contacting surface at the centerline.* Those conditions in Ref. [72] have little to do with tribology. The bottom line is that the authors' Eq. (6) cannot be fundamentally supported and should never be used. That equation affects the authors' parameters at *yielding onset* in their Eqs. (3)–(5).

The authors would have been served better had they reviewed the additional Ref. [6] given below, which was published nearly two decades ago. That reference derives *analytically* all of the relevant parameters at plasticity onset for hemispherical and cylindrical contacts and *specifically* addresses *correctly* how to handle contacting bodies having *dissimilar material properties* without the limitation of the Poisson ratio value over its entire range. That approach is used in Ref. [7] on a whole array of different cases of impacting spheres against flat plates where the bodies have *vast material properties dissimilarities.* Notably, *contact unloading* (the objective of the subject paper [1]) is implicitly incorporated in the *restitution phase* in Ref. [7]. The accuracies of the results in Ref. [7] vary from *excellent to perfect* when compared to finite element analysis and experimental results obtained independently by Higgs et al. [8–10]. To benefit the reader, the following recaps the underlying fundamentals. The discussion below signifies S as the strength, while σ signifies the stress.

When two contacting bodies are ductile and have dissimilar material properties—suppose that the yield strengths, $S_{y1} \neq S_{y2}$, etc.—then it is *imperative* to use the procedure detailed by Green [6] to calculate the critical parameters (significant errors might ensue otherwise). Briefly, in normal elastic contacts, Green [6] defined $C = p_o/\sigma_{e\text{-max}}$ to be the ratio between the maximum Hertzian contact pressure, p_o, and the maximum von Mises stress,[1] $\sigma_{e\text{-max}}$ (which transpires in normal three-dimensional contact under the surface along the centerline). *That ratio solely depends upon the Poisson ratio*. A curve-fit is given in Ref. [3] to a numerical solution of a transcendental equation, rendering for hemispherical (3D) contacts, $C(\nu) = 1.295\exp(0.736\nu)$, while in Ref. [6] a slightly higher fidelity expression is provided, $C(\nu) = 1.30075 + 0.87825\ \nu + 0.54373\ \nu^2$. In *elastic* Hertzian contacts, the two bodies share the same contact area and the same maximum pressure, p_o. Abiding by the von Mises criterion,[2] $\sigma_{e\text{-max}} = S_y$ at yielding onset, the product $C(\nu)\cdot S_y$ gives identically the corresponding *critical contact pressure*, $p_{oy} \equiv C(\nu)\cdot S_y$, that would initiate yielding for that body. For dissimilar materials, Green [6] instructs to pick the *smallest* between two *maximum contact pressure possibilities* to decide which of the contacting bodies yields first. Hence

$$(p_{oy})_{\min} \equiv CS_y = \min\left[C(\nu_1)S_{y1},\ C(\nu_2)S_{y2}\right] \tag{1}$$

where

$$C(\nu) = 1.295\exp(0.736\nu) \tag{2}$$

Equations (1) and (2) are now used to calculate the critical interference, load, and contact radius, which are also given in Ref. [6]. In the subject paper's notation these are

$$\delta_y = \left(\frac{\pi (p_{oy})_{\min}}{2E^*}\right)^2 R = \left(\frac{\pi(CS_y)}{2E^*}\right)^2 R \tag{3}$$

$$F_y = \frac{1}{6}\left(\frac{R}{E^*}\right)^2 \left[\pi (p_{oy})_{\min}\right]^3 = \frac{1}{6}\left(\frac{R}{E^*}\right)^2 \left[\pi(CS_y)\right]^3 \tag{4}$$

$$a_y = \frac{\pi (p_{oy})_{\min}}{2E^*} R = \frac{\pi(CS_y)}{2E^*} R \tag{5}$$

where R is the composite radius calculated by $1/R = 1/R_1 + 1/R_2$, and E^* is the equivalent modulus of elasticity. Emphasizing,

[1]The reader ought to be aware that the equivalent von Mises stress at any material point is: (1) a theoretical stress (i.e., it is not a physical stress) and that via the Poisson ratio it is proportional to the distortion strain energy (and thus it is always positive), (2) obtained solely from the theory of elasticity, and (3) propositioned as a theory for yielding onset (which, of course, is a widely accepted theory today). There is no plasticity involved in the derivation of the distortion-energy (von Mises) yielding theory at all. The critical values are, therefore, free of any plasticity considerations. If the contact load does not exceed the critical value, and is then removed, the contacting bodies shall completely restore their original geometries (shapes) unblemished.

[2]See Note 1.

Contributed by the Tribology Division of ASME for publication in the Journal of Tribology. Manuscript received August 11, 2022; final manuscript received October 31, 2022; published online November 23, 2022. Assoc. Editor: Farshid Sadeghi.

C and S_y always appear grouped in a product to signify the minimum value of the maximum contact pressure that brings upon yielding onset, $(p_{oy})_{min} \equiv (CS_y)$, occurring in *either* material 1 or material 2 according to Eq. (1). Hence, the current Eqs. (3)–(5) given here should replace the corresponding equations in Ref. [1], while current Eqs. (1) and (2) should be used instead of the unfounded Eq. (6) in Ref. [1] (and, of course, in other works as well). Note that all parameters in Eqs. (1)–(5) account accurately for the Poisson ratios regardless of their values or ranges.

Another issue is the authors' usage of ratios such as E^*/S_y throughout the paper (see Table 1 and other locations in the paper). That reciprocal of that ratio denotes an equivalent strain at yielding onset. That aspect is again discussed in Refs. [3,7]. Following the same logic as in Eq. (1), we have

$$\varepsilon_y^* = \begin{cases} S_{y1}/E^* & \leftrightarrow \quad C(\nu_1)S_{y1} \le C(\nu_2)S_{y2} \\ S_{y2}/E^* & \leftrightarrow \quad C(\nu_1)S_{y1} > C(\nu_2)S_{y2} \end{cases} \tag{6}$$

The subject paper [1] uses its own critical parameter definitions to normalize and present results. Using the *correct* parameters given herein is bound to alter to some extent the presence of the outcomes. A considerable effort had been invested in the paper [1]. The subject of contact unloading rightfully deserves such an effort. The purpose of this discussion is to bring attention to the inaccuracies in the definitions used, as they are based on postulations and unfounded theories. There is the *correct* theory, and then there are all other "viewpoints." In summary, based on sound analytical foundations in Ref. [6], this discussion explains and recasts the *correct* definitions of the critical parameters.

Conflict of Interest

There is no conflicts of interest.

Data Availability Statement

The data sets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

References

[1] Chen, C., Wang, Q., Wang, H., Ding, H., Hu, W., Xie, W., Weng, P., Jiang, L., and Yin, X., 2022, "Contact Unloading Behaviors of Elastic-Power-Law Strain Hardening Material Considering Indenter Elasticity Effect," ASME J. Tribol., **144**(12), p. 121501.

[2] Chang, W. R., Etsion, I., and Bogy, D. B., 1987, "An Elastic-Plastic Model for the Contact of Rough Surfaces," ASME J. Tribol., **109**(2), pp. 257–263.

[3] Jackson, R. L., and Green, I., 2005, "A Finite Element Study of Elasto-plastic Hemispherical Contact Against a Rigid Flat," ASME J. Tribol., **127**(2), pp. 343–354.

[4] Green, I., 2018, "Discussion: 'Extended Greenwood-Williamson Models for Rough Spheres' (Zhao, T., and Feng, Y. T., 2018, ASME J. Appl. Mech., 85(10), p. 101007)," ASME J. Appl. Mech., **85**(11), p. 115501.

[5] Ghaednia, H., Wang, X., Saha, S., Xu, Y., Sharma, A., and Jackson, R. L., 2017, "A Review of Elastic–Plastic Contact Mechanics," ASME Appl. Mech. Rev., **69**(6).

[6] Green, I., 2005, "Poisson Ratio Effects and Critical Values in Spherical and Cylindrical Hertzian Contacts," Appl. Mech. Eng., **10**(3), p. 451.

[7] Green, I., 2022, "The Prediction of the Coefficient of Restitution Between Impacting Spheres and Finite Thickness Plates Undergoing Elastoplastic Deformations and Wave Propagation," Nonlinear Dyn., **109**(4), pp. 2443–2458.

[8] Marinack, M. C., Musgrave, R. E., and Higgs, C. F., 2013, "Experimental Investigations on the Coefficient of Restitution of Single Particles," Tribol. Trans., **56**(4), pp. 572–580.

[9] Patil, D., and Fred Higgs, I,C, 2017, "Critical Plate Thickness for Energy Dissipation During Sphere–Plate Elastoplastic Impact Involving Flexural Vibrations," ASME J. Tribol., **139**(4).

[10] Patil, D., and Fred Higgs, C., 2018, "Experimental Investigations on the Coefficient of Restitution for Sphere–Thin Plate Elastoplastic Impact," ASME J. Tribol., **140**(1), p. 011406.