

“The Jackson-Green (JG) contact model”  
By Itzhak Green

Jackson, R. L., and Green, I., “A Finite Element Study of Elasto-plastic Hemispherical Contact Against a Rigid Flat,” **ASME Trans., Journal of Tribology**, Vol. 127, No. 2, (April 2005), 343-354.

Since its publication in 2005, the paper has offered a prominent model for elastoplastic hemispherical contact. While at the time other models have existed too (e.g., the Kogut-Etsion, or KE model), they have all had shortcomings that cannot be unappreciated or overlooked (see the above paper for much details). Notably, JG show that “hardness” should never be used as a material property in this field of contact mechanics (because the so-called “hardness” changes with the deformation). Instead the JG model uses the yield strength, a material property that is unambiguous.

The issue of how to analyze two contacting bodies having dissimilar material properties has been fully addressed in another paper:

Green, I., “Poisson Ratio Effects and Critical Values in Spherical and Cylindrical Hertzian Contacts,” **Int. Journal of Applied Mechanics**, Vol. 10, No. 3, (October 2005) 451-462.

**No other model** handles that issue correctly.

Yet, in another paper that builds upon the JG model, the coefficient of restitution of impacting hemispheres is calculated:

Jackson, R., Green, I., and Marghitu, D., “Predicting the coefficient of restitution of impacting elastic-perfectly plastic spheres,” **Journal of Nonlinear Dynamics**, Vol. 60, No. 3 (May 2010), 217-229.

That paper modified the expression for  $H_G$  (termed the “geometrical hardness” or “average pressure”) to capture its decline also at extreme plasticity (a condition that should not be occurring or tolerated normally in functioning tribological systems). Notably, Dr. Kogut (of the KE model) joined Dr. Jackson (of the JG model) to modify the JG model and the behavior of  $H_G$  under such extreme plasticity:

Wadwalkar, S, Jackson, R., Kogut, L., 2010. "A study of the elastic-plastic deformation of heavily deformed spherical contacts," Proceedings of The Institution of Mechanical Engineers Part J-journal of Engineering Tribology, Vol. 208-210, No. 1 (J10):1-12, DOI: 10.1243/13506501JET763.

The KE model not only is unsuitable under moderate plasticity (while also having issues at the transition from elasticity to elastic-plastic region), it cannot be similarly corrected.

From time to time others have developed or are about to develop "competing" contact models comparing their results to the JG model (as well as to others). Whenever there's a great discrepancy between such models and the JG model, there's invariably some sort of an issue, error, or interpretation of the JG model. Hence, I have decided to once and for all post a computer code (in Mathematica), so that everyone can use, or replicate the calculations. The attached is the code in Mathematica syntax including a PDF file in case you just need a printout.